GROWTH RATES OF AMENABLE GROUPS

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Abstract. Let F_m be a free group with m generators and let R be its normal subgroup such that F_m/R projects onto $\mathbb{Z}$. We give a lower bound for the growth rate of the group F_m/R' (where R' is the derived subgroup of R) in terms of the length $\rho = \rho(R)$ of the shortest non-trivial relation in R. It follows that the growth rate of F_m/R' approaches $2m-1$ as ρ approaches infinity. This implies that the growth rate of an m-generated amenable group can be arbitrarily close to the maximum value $2m-1$. This answers an open question of P. de la Harpe. We prove that such groups can be found in the class of abelian-by-nilpotent groups as well as in the class of virtually metabelian groups.

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