METRICS ON DIAGRAM GROUPS AND UNIFORM EMBEDDINGS IN A HILBERT SPACE

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Abstract. We give first examples of finitely generated groups having an intermediate, with values in $(0, 1)$, Hilbert space compression (which is a numerical parameter measuring the distortion required to embed a metric space into Hilbert space). These groups include certain diagram groups. In particular, we show that the Hilbert space compression of Richard Thompson's group F is equal to $1/2$, the Hilbert space compression of $\mathbb{Z} \wr \mathbb{Z}$ is between $1/2$ and $3/4$, and the Hilbert space compression of $\mathbb{Z} \wr (\mathbb{Z} \wr \mathbb{Z})$ is between 0 and $1/2$. In general, we find a relationship between the growth of H and the Hilbert space compression of $\mathbb{Z} \wr H$.

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