AN ALGORITHM DETECTING DEHN PRESENTATIONS.

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ABSTRACT. An algorithm is given detecting whether or not a finite presentation of a group is a Dehn presentation (i.e. admitting Dehn's algorithm for the word problem) with a certain condition.

Because being word hyperbolic is a Markov property of groups there cannot exist an effective procedure for determining if a finitely presented group admits a Dehn presentation (see, for example, [9]). However, there may exist an algorithm to decide whether a finite presentation of a group is a Dehn presentation. In this article we prove a result in this direction.

Definition 1. Let $\frac{1}{2} \leq \alpha < 1$. We call a presentation $\langle \mathcal{X} \mid \mathcal{R} \rangle$ of a group G an *α-Dehn presentation* if any non-empty freely reduced word w representing the identity in G contains, as a subword, a word u which is also a subword of a cyclic shift of some $r \in \mathcal{R}^{\pm 1}$ with $|u| > \alpha |r|$.

A Dehn presentation in the traditional sense is an α-Dehn presentation with $\alpha = 1/2$. Observe also that any α-Dehn presentation is a Dehn presentation.

A Dehn presentation of a group G leads to a known Dehn's algorithm solving the word problem for G. We refer to [5] for an interesting discussion and problems on other Dehn's algorithms and different notions of Dehn presentations.

Our main result is the following.

Theorem. *There exists an algorithm determining whether or not a finite presentation of a group is an α-Dehn presentation for some $\frac{3}{4} \leq \alpha < 1$.*

Note that a group G given by a finite Dehn presentation is word hyperbolic [6, 2.3], the word problem for G is solvable in linear time by Dehn's algorithm [4], and the hyperbolicity constant for G can be calculated, see for example subsection 1.2.

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