

# QUANTIFYING METRIC APPROXIMATIONS OF DISCRETE GROUPS

GOULNARA ARZHANTSEVA AND PIERRE-ALAIN CHERIX

ABSTRACT. We introduce and systematically study a profile function whose asymptotic behavior quantifies the dimension or the size of a metric approximation of a finitely generated group  $G$  by a family of groups  $\mathcal{F} = \{(G_\alpha, d_\alpha, k_\alpha, \varepsilon_\alpha)\}_{\alpha \in I}$ , with each group  $G_\alpha$  equipped with a bi-invariant metric  $d_\alpha$  and a dimension  $k_\alpha$ , for strictly positive real numbers  $\varepsilon_\alpha$  such that  $\inf_\alpha \varepsilon_\alpha > 0$ . Through the notion of residually amenable profile we introduce, our approach generalizes classical isoperimetric (or Følner) profiles of amenable groups and recently introduced functions quantifying residually finite groups. Our viewpoint is much more general and covers hyperlinear and sofic approximations as well as many other metric approximations such as weakly sofic, weakly hyperlinear, and linear sofic approximations.

UNIVERSITÄT WIEN, FAKULTÄT FÜR MATHEMATIK, OSKAR-MORGENSTERN-PLATZ 1, 1090 WIEN, AUSTRIA.  
*E-mail address:* `goulnara.arzhantseva@univie.ac.at`

UNIVERSITÉ DE GENÈVE, SECTION DE MATHÉMATIQUES, 2-4 RUE DU LIÈVRE, CASE POSTALE 64, 1211 GENÈVE 4, SWITZERLAND.

*E-mail address:* `pierre-alain.cherix@unige.ch`

---

2010 *Mathematics Subject Classification.* 20E26, 20F69, 20C99.

*Key words and phrases.* Residually finite groups, sofic and hyperlinear groups, metric ultraproducts, amenable groups.

This research was partially supported by the European Research Council (ERC) grant of Goulnara Arzhantseva, “ANALYTIC” grant agreement no. 259527.