UNIFORM NON-AMENABILITY

G.N. ARZHANTSEVA, J. BURILLO, M. LUSTIG, L. REEVES, H. SHORT, AND E. VENTURA

Abstract. For any finitely generated group G an invariant $\mathrm{Føl}\, G \geq 0$ is introduced which measures the "amount of non-amenability" of G. If G is amenable, then $\mathrm{Føl}\, G = 0$. If $\mathrm{Føl}\, G > 0$, we call G *uniformly non-amenable*. We study the basic properties of this invariant; for example, its behaviour when passing to subgroups and quotients of G. We prove that the following classes of groups are uniformly non-amenable: non-abelian free groups, non-elementary word-hyperbolic groups, large groups, free Burnside groups of large enough odd exponent, and groups acting acylindrically on a tree. Uniform non-amenability implies uniform exponential growth. We also exhibit a family of non-amenable groups (in particular including all non-solvable Baumslag-Solitar groups) which are not uniformly non-amenable, that is, they satisfy $\mathrm{Føl}\, G = 0$. Finally, we derive a relation between our uniform Følner constant and the uniform Kazhdan constant with respect to the left regular representation of G.

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University of Geneva, CH-1211 Geneva 24, Switzerland
E-mail address: goulnara.arjantseva@math.unige.ch

Universitat Politècnica de Catalunya, Avda. del Canal Olímpic, 08860 Castelldefels (Barcelona), Spain
E-mail address: burillo@mat.upc.es

LATP, UMR CNRS 6632, FRUMAM, Université d'Aix-Marseille III, Avenue Escadrille Normandie-Niemen, 13397 Marseille 20, France
E-mail address: martin.lustig@univ.u-3mrs.fr

LATP, UMR CNRS 6632, FRUMAM, Université d'Aix-Marseille III, Avenue Escadrille Normandie-Niemen, 13397 Marseille 20, France

LATP, UMR CNRS 6632, FRUMAM, University de Provence, CMI, 39 rue Joliot Curie, 13453 Marseille 13, France
E-mail address: hamish.short@cmi.univ-mrs.fr

Universitat Politècnica de Catalunya, Av. Bases de Manresa 61 - 73, 08240 Manresa (Barcelona), Spain
E-mail address: enric.ventura@upc.es