# Summary and comments to my list of publications

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[57] G. Arzhantseva, M. Steenbock, *Rips construction without unique product*, Pacific Journal of Mathematics, (2022), in press, arXiv:1407.2441.

Given a finitely presented group $Q$, we produce a short exact sequence $1 \to N \hookrightarrow G \twoheadrightarrow Q \to 1$ such that $G$ is a torsion-free hyperbolic group without the unique product property and $N$ is without the unique product property and has Kazhdan's Property (T). Varying $Q$ yields a wide diversity of concrete examples of hyperbolic groups without the unique product property. We also note, as an application of Ol'shanskii's construction of torsion-free Tarski monsters, the existence of torsion-free Tarski monster groups without the unique product property.

[56] G. Arzhantseva, A. Biswas, *Logarithmic girth expander graphs of* $SL_n(\mathbb{F}_p)$,
Journal of Algebraic Combinatorics, 56 (2022), 691–723, 10.107/s10801-022-01128-z

We provide an explicit construction of finite 4-regular graphs $(\Gamma_k)_{k\in\mathbb{N}}$ with girth $\Gamma_k \to \infty$ as $k \to \infty$ and $\frac{\operatorname{diam}\Gamma_k}{\operatorname{girth}\Gamma_k} \leqslant D$ for some $D > 0$ and all $k \in \mathbb{N}$. For each fixed dimension $n \geqslant 2$, we find a pair of matrices in $SL_n(\mathbb{Z})$ such that (i) they generate a free subgroup, (ii) their reductions $\bmod\, p$ generate $SL_n(\mathbb{F}_p)$ for all sufficiently large primes $p$, (iii) the corresponding Cayley graphs of $SL_n(\mathbb{F}_p)$ have girth at least $c_n \log p$ for some $c_n > 0$. Relying on growth results (with no use of expansion properties of the involved graphs), we observe that the diameter of those Cayley graphs is at most $O(\log p)$. This gives infinite sequences of finite 4-regular Cayley graphs of $SL_n(\mathbb{F}_p)$ as $p \to \infty$ with large girth and bounded diameter-by-girth ratio. These are the first explicit examples in all dimensions $n \geqslant 2$ (all prior examples were in $n = 2$). Moreover, they happen to be expanders. Together with Margulis' and Lubotzky-Phillips-Sarnak's classical constructions, these new graphs are the only known explicit logarithmic girth Cayley graph expanders.

[55] G. N. Arzhantseva, M. Hagen, *Acylindrical hyperbolicity of cubical small-cancellation groups*, Algebraic & Geometric Topology, 22(5) (2022), 2007–2078, 10.2140/agt.2022.22.2007

We provide an analogue of Strebel's classification of geodesic triangles in classical $C'(1/6)$ groups for groups given by Wise's cubical presentations satisfying sufficiently strong metric cubical small cancellation conditions. Using our classification, we give conditions guaranteeing that a cubical small cancellation group is acylindrically hyperbolic.

[54] G.N. Arzhantseva, S. Gal, *On approximation properties of semidirect products of groups*, Annales mathematiques Blaise Pascal, 27(1) (2020), 125-130, doi: 10.5802/ambp.386.

Let $\mathcal{R}$ be a class of groups closed under taking semidirect products with finite kernel and fully residually $\mathcal{R}$–groups. We prove that $\mathcal{R}$ contains all $\mathcal{R}$–by–{ finitely generated residually finite} groups. It follows that a semidirect product of a finitely generated residually finite group with a surjunctive group is surjunctive. This remained unknown even for direct products of a surjunctive group with the integers $\mathbb{Z}$.

[53] G. N. Arzhantseva, F. Berlai, M. Finn-Sell, L. Glebsky, *Unrestricted wreath products and sofic groups*,International Journal of Algebra and Computation, 29(02) (2019), 343-355, doi:10.1142/S021819671950005X.

We show that the unrestricted wreath product of a sofic group by an amenable group is sofic. We use this result to present an alternative proof of the known fact that any group extension with sofic kernel and amenable quotient is again a sofic group. Our approach exploits the famous Kaloujnine-Krasner theorem and extends, with an additional argument, to hyperlinear-by-amenable groups.

[52] G. N. Arzhantseva, Ch. Cashen, *Cogrowth for group actions with strongly contracting elements*, Ergodic Theory and Dynamical Systems, 40(7) (2020), 1738-1754, doi:10.1017/etds.2018.123.

Let $G$ be a group acting properly by isometries and with a strongly contracting element on a geodesic metric space. Let $N$ be an infinite normal subgroup of $G$, and let $\delta_N$ and $\delta_G$ be the growth rates of $N$ and $G$ with respect to the pseudo-metric induced by the action. We prove that if $G$ has purely exponential growth with respect to the pseudo-metric then $\delta_N/\delta_G > 1/2$. Our result applies to suitable actions of hyperbolic groups, right-angled Artin groups and other CAT(0) groups, mapping class groups, snowflake groups, small cancellation groups, etc. This extends Grigorchuk's original result on free groups with respect to a word metrics and a recent result of Jaerisch, Matsuzaki, and Yabuki on groups acting on hyperbolic spaces to a much wider class of groups acting on spaces that are not necessarily hyperbolic.

[51] G. N. Arzhantseva, L. Paunescu, Constraint metric approximations and equations in groups, Journal of Algebra, 516 (2018), 329-351, doi:10.1016/j.jalgebra.2018.09.007.

We introduce notions of a constraint metric approximation and of a constraint stability of a metric approximation. This is done in the language of group equations with coefficients. We give an example of a group which is not constraintly sofic. In building it, we find a sofic representation of free group with trivial commutant among extreme points of the convex structure on the space of sofic representations.

We consider the centralizer equation in permutations as an instance of this new general setting. We characterize permutations $p \in S_k$ whose centralizer is stable in permutations with respect to the normalized Hamming distance, that is, a permutation which almost centralizes $p$ is near a centralizing permutation. This answers a question of Gorenstein-Sandler-Mills (1962).

[50] G. N. Arzhantseva, C. Drutu, *Geometry of infinitely presented small cancellation groups and quasi-homomorphisms*, Canadian Journal of Mathematics, 71(5) (2019), 997-1018, doi:10.4153/CJM-2018-036-7.

We study the geometry of infinitely presented groups satisfying the small cancellation condition $C'(1/8)$, and introduce a standard decomposition (called the *criss-cross decomposition*) for the elements of such groups. Our method yields a direct construction of a linearly independent set of power continuum in the kernel of the comparison map between the bounded and the usual group cohomology in degree 2, without the use of free subgroups and extensions.

[49] G. N. Arzhantseva, Ch. Cashen, D. Gruber, D. Hume, *Negative curvature in graphical small cancellation groups*, Groups, Geometry and Dynamics, 13(2) (2019), 579-632, doi:10.4171/GGD/498.

We use the interplay between combinatorial and coarse geometric versions of negative curvature to investigate the geometry of infinitely presented graphical $Gr'(1/6)$ small cancellation groups. In particular, we characterize their "contracting geodesics", which should be thought of as the geodesics that behave hyperbolically.

We show that every degree of contraction can be achieved by a geodesic in a finitely generated group. We construct the first example of a finitely generated group $G$ containing an element $g$ that is strongly contracting with respect to one finite generating set of $G$ and not strongly contracting with respect to another. In the case of classical $C'(1/6)$ small cancellation groups we give complete characterizations of geodesics that are Morse and that are strongly contracting.

We show that many graphical $Gr'(1/6)$ small cancellation groups contain strongly contracting elements and, in particular, are growth tight. We construct uncountably many quasi-isometry classes of finitely generated, torsion-free groups in which every maximal cyclic subgroup is hyperbolically embedded. These are the first examples of this kind that are not subgroups of hyperbolic groups.In the course of our analysis we show that if the defining graph of a graphical $Gr'(1/6)$ small cancellation group has finite components, then the elements of the group have translation lengths that are rational and bounded away from zero.

[48] G. N. Arzhantseva, R. Tessera, *Admitting a coarse embedding is not preserved under group extensions*, International Mathematics Research Notices, 2019 (20) (2019), 6480-6498, doi:10.1093/imrn/rny017.

We construct a finitely generated group which is an extension of two finitely generated groups coarsely embeddable into Hilbert space but which itself does not coarsely embed into Hilbert space. Our construction also provides a new infinite monster group: the first example of a finitely generated group that does not coarsely embed into Hilbert space and yet does not contain a weakly embedded expander.

[47] G. N. Arzhantseva, G. A. Niblo, N. Wright, J. Zhang, *A characterization for asymptotic dimension growth*, Algebraic & Geometric Topology, 18 (2018), 493-524, doi:10.2140/agt.2018.18.493.

We give a characterization for asymptotic dimension growth. We apply it to CAT(0) cube complexes of finite dimension, giving an alternative proof of Wright's result on their finite asymptotic dimension. We also apply our new characterization to geodesic coarse median spaces of finite rank and establish that they have subexponential asymptotic dimension growth. This strengthens a recent result of Špakula and Wright.

[46] G. N. Arzhantseva, Ch. Cashen, D. Gruber, D. Hume, *Characterizations of Morse quasi-geodesics via superlinear divergence and sublinear contraction*, Documenta Mathematica, 22 (2017), 1193-1224, doi:10.25537/dm.2017v22.1193-1224.

We introduce and begin a systematic study of sublinearly contracting projections. We give two characterizations of Morse quasi-geodesics in an arbitrary geodesic metric space. One is that they are sublinearly contracting; the other is that they have completely superlinear divergence. We give a further characterization of sublinearly contracting projections in terms of projections of geodesic segments.

[45] G. N. Arzhantseva, L. Paunescu, *Linear sofic groups and algebras*, Transactions of the American Mathematical Society 369 (2017), 2285-2310. doi:10.1090/tran/6706.

The major goal of our paper is to introduce the concept of soficity for algebras. We shall approximate our algebras by matrix algebras endowed with a distance provided by the rank. Two matrices are close in this distance if they are equal, as linear transformations, on a large subspace. This is in essence similar to the Hamming distance. Therefore, we call the corresponding algebras (and groups, respectively) *linear sofic*.

We introduce and systematically study linear sofic groups and linear sofic algebras. This generalizes amenable and LEF groups and algebras. We prove that a group is linear sofic if and only if its group algebra is linear sofic. We show that linear soficity for groups is a priori weaker than soficity but stronger than weak soficity. We also provide an alternative proof of a result of Elek and Szabo which states that sofic groups satisfy Kaplansky's direct finiteness conjecture.

[44] G. N. Arzhantseva, Ch. Cashen, J. Tao, *Growth tight actions*, Pacific Journal of Mathematics, 278(1) (2015), 1-49, doi:10.2140/pjm.2015.278.1.

We introduce and systematically study the concept of a growth tight action. This generalizes growth tightness for word metrics as initiated by Grigorchuk and de la Harpe. Given a finitely generated, non-elementary group $G$ acting on a $G$–space $\mathcal{X}$, we prove that if $G$ contains a strongly contracting element and if $G$ is not too badly distorted in $\mathcal{X}$, then the action of $G$ on $\mathcal{X}$ is a growth tight action. It follows that if $\mathcal{X}$ is a cocompact, relatively hyperbolic $G$–space, then the action of $G$ on $\mathcal{X}$ is a growth tight action. This generalizes all previously known results for growth tightness of cocompact actions: every already known example of a group that admits a growth tight action and has some infinite, infinite index normal subgroups is relatively hyperbolic, and, conversely, relatively hyperbolic groups admit growth tight actions. This also allows us to prove that many CAT(0) groups, including flip-graph-manifold groups and many Right Angled Artin Groups, and snowflake groups admit cocompact, growth tight actions. These provide first examples of non relatively hyperbolic groups admitting interesting growth tight actions. Our main result applies as well to cusp uniform actions on hyperbolic spaces and to the action of the mapping class group on Teichmüller space with the Teichmüller metric. Towards the proof of our main result, we give equivalent characterizations of strongly contracting elements and produce new examples of group actions with strongly contracting elements.

[43] G. N. Arzhantseva, L. Paunescu, Almost commuting permutations are near commuting permutations, Journal of Functional Analysis, (2015), doi:10.1016/j.jfa.2015.02.013.

A famous open problem asks whether or not two almost commuting matrices are necessarily close to two exactly commuting matrices. This is considered independently of the matrix sizes and the terms "almost" and "close" are specified with respect to a given norm. The problem is renowned thanks to its connection to physics, originally noticed by von Neumann in his approach to quantum mechanics. The commutator equation being an example, the existence of exactly commuting matrices near almost commuting matrices can be viewed in a wider context of *stability* conceived by Ulam: an equation is stable if an almost solution (or a solution of the corresponding inequality) is near an exact solution.

We prove that the commutator is stable in permutations endowed with the Hamming distance, that is, two permutations that almost commute are near two commuting permutations. Our result extends to $k$-tuples of almost commuting permutations, for any given $k$, and allows restrictions, for instance, to even permutations.

[42] G. N. Arzhantseva, D. Osajda, *Infinitely presented small cancellation groups have Haagerup property*, Journal of Topology and Analysis, (2015), doi:10.1142/S1793525315500144.

A second countable, locally compact group $G$ has the Haagerup property (or $G$ is *a-T-menable* in the sense of Gromov) if it possesses a proper continuous affine isometric action on a Hilbert space. Higson and Kasparov established the strong Baum–Connes conjecture (and, hence, the Baum–Connes conjecture with coefficients) for groups with the Haagerup property.

We prove the Haagerup property (= Gromov's a-T-menability) for finitely generated groups defined by infinite presentations satisfying the $C'(1/6)$-small cancellation condition. We deduce that these groups are coarsely embeddable into a Hilbert space and that the strong Baum- Connes conjecture holds for them. The result is a first nontrivial advancement in understanding groups with such properties among infinitely presented non-amenable direct limits of hyperbolic groups. The proof uses the structure of a space with walls introduced by Wise. As the main step we show that $C'(1/6)$-complexes satisfy the linear separation property.

[41] G. N. Arzhantseva, R. Tessera, *Relative expanders*, Geometric and Functional Analysis [GAFA], 25 (2015), 317–341, doi:10.1007/s00039-015-0316-9.

A well-known obstruction for a metric space to coarsely embed into a Hilbert space is to admit a weakly embedded expander. In this paper, we address the following question:

*Given a metric space which does not embed coarsely into a Hilbert space, does it necessary contain a weakly embedded expander?*

We provide a strongly negative answer. We exhibit a finitely generated group $G$ and a sequence of finite index normal subgroups $N_n \trianglelefteq G$ such that for every finite generating subset $S \subseteq G$, the sequence of finite Cayley graphs $(G/N_n, S)$ does not coarsely embed into any $L^p$-space for $1 \leq p < \infty$ (moreover, into any uniformly curved Banach space), and yet admits no weakly embedded expander. The reason why our examples do not coarsely embed is a new phenomenon called relative expansion, which we define in terms of Poincaré inequalities.

[40] G. N. Arzhantseva, *Asymptotic approximations of finitely generated groups*, in Research Perspectives CRM Barcelona-Fall 2012 (Trends in Mathematics), Birkhäuser, Basel, vol. 1, 2014, 5–12. doi:10.1007/978-3-319-05488-9.

The concept of approximation is ubiquitous in mathematics. A classical idea is to approximate objects of interest by ones simpler to investigate, and which have the required characteristics in order to reflect properties and behaviour of the elusive objects we started with.

Looking for approximation in geometric group theory, first we adapt this fundamental approach. We discuss both its well-established appearance in *residual properties* of groups and its recent manifestation via *metric approximations* of groups such as sofic and hyperlinear approximations. We focus on approximations of Gromov hyperbolic groups, comment open problems, and suggest a conjecture in this setting. Then we turn over this classical way and initiate the study of approximations by groups usually known as being not so elementary to investigate. This allows to see that many interesting groups (still unknown to have algebraic or metric approximations) admit this new type of approximation which we call *asymptotic approximations*. We give many examples of asymptotically sofic / hyperlinear groups, as well as of asymptotically non-residually finite groups. In particular, we provide the first examples of infinite simple asymptotically residually finite (resp. asymptotically amenable) groups with Kazhdan's property (T).

[39] G. N. Arzhantseva, J.-F. Lafont, A. Minasyan,*Isomorphism versus commensurability for a class of finitely presented groups*, Journal of Group Theory, 17(2) (2014), 361–378, doi:10.1515/jgt-2013-0050.

The purpose of this paper is to study the relative algorithm complexities of the following two major group theoretical decision problems: the isomorphism problem and the commensurability problem.

We construct a class of finitely presented groups where the isomorphism problem is solvable but the commensurability problem is unsolvable. Conversely, we construct a class of finitely presented groups within which the commensurability problem is solvable but the isomorphism problem is unsolvable. These are the first examples of such a contrastive complexity behavior with respect to the isomorphism problem.

[38] G. N. Arzhantseva, E. Guentner, J. Spakula, *Coarse non-amenability and coarse embeddings*, Geometric and Functional Analysis [GAFA], 22(1) (2012), 22–36, doi:10.1007/s00039-012-0145-z.

We construct the first example of a coarsely non-amenable (= without Guoliang Yu's property A) metric space with bounded geometry which coarsely embeds into a Hilbert space. Our space $X = \coprod_{n=0}^{\infty} X_n$ is made of the Cayley graphs $X_n$ of certain finite quotients of the free group $\mathbb{F}_2$. We describe $X_n$ in a graph-theoretical way as a tower of successive $\mathbb{Z}/2\mathbb{Z}$-homology covers, starting with the "figure eight" graph. Next, we define a wall structure on each $X_n$, which gives rise to a wall metric on the graphs $X_n$. We show that these graphs, endowed with the wall metric, coarsely embed into a Hilbert space (uniformly).

Our construction and its variants have strong applications, see, for example, our work on relative expanders [41] and our construction of a coarsely embeddable group which is not $C^*$- exact [12].

[37] G. N. Arzhantseva, E. Guentner, *Coarse non-amenability and covers with small eigenvalues*, Mathematische Annalen, 354(3) (2012), 863–870, doi:10.1007/s00208-011-0759-8.

Given a closed Riemannian manifold $M$ and a (virtual) epimorphism $\pi_1(M) \twoheadrightarrow \mathbb{F}_2$ of the fundamental group onto a free group of rank 2, we construct a tower of finite sheeted regular covers $\{M_n\}_{n=0}^{\infty}$ of $M$ such that $\lambda_1(M_n) \to 0$ as $n \to \infty$. This is the first example of such a tower which is not obtainable up to uniform quasi-isometry (or even up to uniform coarse equivalence) by the previously known methods where $\pi_1(M)$ is supposed to surject onto an amenable group.

[36] G. N. Arzhantseva, M. Bridson, T. Januszkiewicz, I. Leary, A. Minasyan, J. Swiatkowski, *Infinite groups with fixed point properties*, Geometry and Topology, 13 (2009), 1229–1263, doi:10.2140/gt.2009.13.1229.

We construct finitely generated groups with strong fixed point properties. Let $\mathcal{X}_{ac}$ be the class of Hausdorff spaces of finite covering dimension which are mod-$p$ acyclic for at least one prime $p$. We produce the first examples of infinite finitely generated groups $Q$ with the property that for any action of $Q$ on any $X \in \mathcal{X}_{ac}$, there is a global fixed point. Moreover, $Q$ may be chosen to be simple and to have Kazhdan's property (T). We construct a finitely presented infinite group $P$ that admits no non-trivial action by diffeomorphisms on any smooth manifold in $\mathcal{X}_{ac}$. In building $Q$, we exhibit new families of hyperbolic groups: for each $n \geq 1$ and each prime $p$, we construct a non-elementary hyperbolic group $G_{n,p}$ which has a generating set of size $n+2$, any proper subset of which generates a finite $p$-group.

[35] G. N. Arzhantseva, C. Druţu, and M. Sapir, *Compression functions of uniform embeddings of groups into Hilbert and Banach spaces*, Journal für die Reine und Angewandte Mathematik, [Crelle's Journal], 633 (2009), 213–235, doi:10.1515/CRELLE.2009.066.

We construct finitely generated groups with arbitrary prescribed Hilbert space compression $\alpha \in [0, 1]$. This answers a question of E. Guentner and G. Niblo. For a large class of Banach spaces $\mathcal{E}$ (including all uniformly convex Banach spaces), the $\mathcal{E}$–compression of these groups coincides with their Hilbert space compression. Moreover, the groups that we construct have asymptotic dimension at most 2, hence they are exact. In particular, the first examples of groups that are uniformly embeddable into a Hilbert space (moreover, of finite asymptotic dimension and exact) with Hilbert space compression 0 are given. These groups are also the first examples of groups with uniformly convex Banach space compression 0.

[34] G. N. Arzhantseva, V. S. Guba, M. Lustig and J.-Ph. Préaux, *Testing Cayley graph densities*, Annales mathematiques Blaise Pascal 15(2) (2008), 169–221, doi:10.5802/ambp.249.

We present a computer-assisted analysis of combinatorial properties of the Cayley graphs of certain finitely generated groups: Given a group with a finite set of generators, we study the density of the corresponding Cayley graph, that is, the least upper bound for the average vertex degree (= number of adjacent edges) of any finite subgraph. It is known that an $m$-generated group is amenable if and only if the density of the corresponding Cayley graph equals to $2m$. We test amenable and non-amenable groups, and also groups for which amenability is unknown. In the latter class we focus on Richard Thompson's group $F$.

[33] G. N. Arzhantseva, A. Minasyan and D. Osin, *The SQ-universality and residual properties of relatively hyperbolic groups*, Journal of Algebra, **315**(1) (2007), 165-177, doi:10.1016/j.jalgebra.2007.04.029.

We apply methods of generalized small cancellation theory to study the residual properties of relatively hyperbolic groups. In particular, we prove that if a group $G$ is non–elementary and hyperbolic relative to a collection of proper subgroups, then $G$ is SQ–universal. We apply our method to show that if $G$ is finitely presented then it possesses a lot of finitely presented quotients having "monster"-like group-theoretic properties.

We show that any two finitely generated non-elementary relatively hyperbolic groups have a common non-elementary relatively hyperbolic quotient. This became a universal tool for producing new groups with "wild" structure, cf. [36]. Another result from this paper answers a long-standing question by Fine and Tretkoff pursued since 1979.

[32] G. N. Arzhantseva, Z. Šunić, *Construction of elements in the closure of Grigorchuk group*, Geometriae Dedicata, **124**(1) (2007), 17–26.

The first Grigorchuk group has many remarkable properties (e.g. it is a finitely generated infinite torsion group; it is amenable but not elementary amenable and has intermediate growth). This group can be viewed as a group of automorphisms of the binary rooted tree $\mathcal{T}$. We describe constraints that need to be satisfied "near the top" of the portraits of the elements in Grigorchuk group. These constraints, if satisfied by the portraits of all sections of some binary tree automorphism, guarantee that this automotphism belongs to the closure of Grigorchuk group in the pro-finite group of binary tree automorphisms. This gives an effective way to construct all elements of the closure. This answers a question of Grigorchuk. We also build elements in the closure that do not belong to the group.

[31] G. N. Arzhantseva, A. Minasyan, *Relatively hyperbolic groups are $C^*$-simples*, Journal of Functional Analysis, **243**(1), (2007), 345-351, doi:10.1016/j.jfa.2006.06.003.

A countable group G is $C^*$-simple if its reduced $C^*$-algebra is simple. The $C^*$-simplicity can be regarded as a strong form of non-amenability. In 1975 Powers established the $C^*$-simplicity of non-abelian free groups. Later, many other examples of $C^*$-simple groups where found in geometry and in group theory. In this paper, we characterize relatively hyperbolic groups whose reduced $C^*$-algebra is simple as those, which have no non-trivial finite normal subgroups. More precisely, we show that in a relatively hyperbolic group $G$ without finite normal subgroups, given any finite subset $F$, there exists $g_0 \in G$ of infinite order such that, for any $f \in F$, the subgroup of $G$ generated by $g_0$ and $f$ is isomorphic to the free product of the cyclic subgroups $\langle g_0 \rangle$ and $\langle f \rangle$. An interesting corollary of the main result is that the amenable radical of non-elementary properly relatively hyperbolic group coinsides with it maximal finite normal subgroup $E(G)$ and the quotient $G/E(G)$ is $C^*$-simple with a unique normalized trace.

Every non-elementary Gromov hyperbolic group is relatively hyperbolic with respect to the family consisting of the trivial subgroup. Therefore our result also describes all $C^*$-simple Gromov hyperbolic groups.

[30] G. N. Arzhantseva, P. de la Harpe and D. Kahrobaei, *The true prosoluble completion of agroup: examples and open problems,* Geometriae Dedicata, **124**(1) (2007), 5-17. doi:10.1007/s10711-006-9103-y.

The true prosoluble completion $P\mathcal{S}(\Gamma)$ of a group $\Gamma$ is the inverse limit of the projective system of soluble quotients of $\Gamma$. Our purpose is to describe examples and to point out some natural open problems. We discuss a question of Grothendieck for profinite completions and its analogue for true prosoluble and true pronilpotent completions.

[29] G. N. Arzhantseva, V. S. Guba, M. V. Sapir, *Metrics on diagram groups and uniform embeddings in a Hilbert space*, Commentarii Mathematici Helvetici, **81**(4) (2006), 911–929, doi:10.4171/CMH/80.

We give first examples of finitely generated groups having an intermediate, with values in $(0,1)$, Hilbert space compression (which is a numerical parameter measuring the distortion required to embed a metric space into Hilbert space). These groups include certain diagram groups. In particular, we show that the Hilbert space compression of Richard Thompson's group $F$ is equal to 1/2, the Hilbert space compression of $\mathbb{Z} \wr \mathbb{Z}$ is between 1/2 and 3/4, and the Hilbert space compression of $\mathbb{Z} \wr (\mathbb{Z} \wr \mathbb{Z})$ is between 0 and 1/2. In general, we find a relationship between the growth of $H$ and the Hilbert space compression of $\mathbb{Z} \wr H$.

Our work has given rise to a new subject: computation of compression functions of coarse embeddings of graphs and groups.

[28] G. N. Arzhantseva, *A dichotomy for subgroups of word hyperbolic groups,* Contemporary Mathematics, **394**, Amer. Math. Soc., Providence, RI, 2006, 1-11.

Let $H$ be a subgroup of a word hyperbolic group $G$. Our main result gives a sufficient condition for $H$ to be free and quasiconvex in $G$. It is an improvement of a result due to Gromov (see [5.3.A] of "Hyperbolic groups").

Given $L > 0$ elements in a word hyperbolic group $G$, there exists a number $M = M(G, L) > 0$ such that at least one of the assertions is true: (i) these elements generate a free and quasiconvex subgroup of $G$; (ii) they are Nielsen equivalent to a system of $L$ elements containing an element of length at most $M$ up to conjugation in $G$.

The constant $M$ is given explicitly. The result is generalized to groups acting by isometries on Gromov hyperbolic spaces. For proof we use a graph method to represent finitely generated subgroups of a group. The technique is of independent interest. In particular, transformations of a labelled graph defined in the paper can be viewed as a generalization of free reductions and Nielsen reductions of tuples of group elements.

[27] G. N. Arzhantseva and I. G. Lysenok, *A lower bound on the growth of word hyperbolic groups,* Journal of the London Mathematical Society, (2) **73** (2006), 109-125, doi:10.1112/S002461070502257X

Apart from several simple examples, no classes of groups are known for which the exponential growth rate achieves its infimum on some generating set. In this paper, we try to do a step towards the proof of the conjecture that, for non-elementary word hyperbolic groups the infimum of the exponential growth rate is achieved on some of its generating sets. Namely, we give a linear lower bound on the exponential growth rate of a non-elementary subgroup of a word hyperbolic group, with respect to the number of generators for the subgroup. As an immediate consequence of this we get another results (see Corollary and Theorem 2) which restrict generating sets to check the property of a group to reach its uniform exponential growth rate.

As observed in [24], an affirmative answer to the conjecture would imply that a non-elementary word hyperbolic group is Hopfian. Note that word hyperbolic groups which are *torsion free* are known to be Hopfian by a result of Z. Sela.

[26] G. N. Arzhantseva, V. S. Guba, L. Guyot, *Growth rates of amenable groups*, Journal of Group Theory, **8** (2005), no. 3, 389–394, doi:10.1515/jgth.2005.8.3.389.

Let $F_m$ be a free group with $m$ generators and let $R$ be its normal subgroup such that $F_m/R$ projects onto $\mathbb{Z}$. We give a lower bound for the growth rate of the group $F_m/R'$ (where $R'$ is the derived subgroup of $R$) in terms of the length $\rho = \rho(R)$ of the shortest nontrivial relation in $R$. It follows that the growth rate of $F_m/R'$ approaches $2m-1$ as $\rho$ approaches infinity. This implies that the growth rate of an $m$-generated amenable group can be arbitrarily close to the maximum value $2m-1$. This answers question 15.4 by P. de la Harpe from the "Kourovka Notebook" (Unsolved problems in group theory). In fact we prove that such groups can be found already in the class of abelian-by-nilpotent groups as well as in the class of finite extensions of metabelian groups.

[25] G. N. Arzhantseva, J. Burillo, M. Lustig, L. Reeves, H. Short, E. Ventura, *Uniform non-amenability,* Advances in Mathematics, **197** (2005), no. 2, 499–522, doi:10.1016/j.aim. 2004.10.013.

For any finitely generated group $G$ an invariant $\mathrm{Føl}\, G \geq 0$ is introduced which measures the "amount of non-amenability" of $G$. If $G$ is amenable, then $\mathrm{Føl}\, G = 0$. If $\mathrm{Føl}\, G > 0$, we call $G$ *uniformly non-amenable*. We study the basic properties of this invariant; for example, its behaviour when passing to subgroups and quotients of $G$. We prove that the following classes of groups are uniformly non-amenable: non-abelian free groups, non-elementary word-hyperbolic groups, large groups, free Burnside groups of large enough odd exponent, and groups acting acylindrically on a tree. Uniform non-amenability implies uniform exponential growth. We also exhibit a family of *non-amenable* groups (in particular including all non-solvable Baumslag-Solitar groups) which are not uniformly non-amenable, that is, they satisfy $\mathrm{Føl}\, G = 0$. Finally, we derive a relation between our uniform Følner constant and the uniform Kazhdan constant with respect to the left regular representation of $G$.

Our paper has initiated a lot of research activity on uniform properties of groups, e.g. uniformly non-amenable measured equivalence relations and a Følner invariant for type $II_1$ factors were discovered in operator algebra.

[24] G. N. Arzhantseva and P.-A. Cherix, *On the Cayley graph of a generic finitely presented group*, Bulletin of the Belgian Mathematical Society, **11** (2004), no. 4, 589–601, doi:10.36045/bbms/1102689123.

We prove that in a certain statistical sense the Cayley graph of almost every finitely presented group with $m \geq 2$ generators contains a subdivision of the complete graph on $l \leq 2m+1$ vertices. In particular, this Cayley graph is non planar and the genus of a generic finitely presented group (that is the minimum of genus of its Cayley graphs taken over all generating sets) is the infinity. We also show that some group constructions preserve the planarity.

[23] G. N. Arzhantseva and I. G. Lysenok, *Growth tightness for word hyperbolic groups,* Mathematische Zeitschrift, **241** (2002), no. 3, 597–611, doi:10.1007/s00209-002-0434-6.

We show that non-elementary word hyperbolic groups are growth tight. This means that, given such a group $G$ and a finite set $A$ of its generators, for any infinite normal subgroup $N$ of $G$, the exponential growth rate of $G/N$ with respect to the natural image of $A$ is strictly less than the exponential growth rate of $G$ with respect to $A$. This gives an affirmative answer to the question about growth tightness of word hyperbolic groups, posed by R. Grigorchuk and P. de la Harpe.

[22] G. N. Arzhantseva and D. V. Osin, *Solvable groups with polynomial Dehn functions*, Transactions of the American Mathematical Society, **354** (2002), 3329–3348, doi:10.1090/S0002-9947-02-02985-9

Given a finitely presented group $H$, finitely generated subgroup $B$ of $H$, and a monomorphism $\psi : B \to H$, we obtain an upper bound of the Dehn function of the corresponding HNN-extension

$G = \langle H, t \mid t^{-1}Bt = \psi(B) \rangle$ in terms of the Dehn function of $H$ and the distortion of $B$ in $G$. Using such a bound, we construct first examples of non-polycyclic solvable groups with polynomial Dehn functions. The constructed groups are metabelian and contain the solvable Baumslag-Solitar groups. In particular, this answers a question posed by Birget, Ol'shanskii, Rips, and Sapir. (This question goes up to the problem of simulating of Turing mashines in groups with a good control of Dehn functions using ideas of the Novikov-Boone construction.)

Note that all finitely presented solvable groups that were known to have polynomial Dehn functions are nilpotent (by results of Gromov, Pittet) or polycyclic (by results of Drutu).

First examples of finitely presented groups $H(q)$ with the Dehn functions $\delta_{H(q)}(n) \simeq n^{10}$ containing the solvable Baumslag-Solitar group $BS(1,q)$, $q > 1$, were obtained by Ol'shanskii and Sapir. However, the groups constructed by them are very distant from solvable ones.

[21] G. N. Arzhantseva, *On quasiconvex subgroups of word hyperbolic groups*,
Geometriae Dedicata, **87** (2001), 191–208, doi:1023/A:1012040207144.

We prove that a quasiconvex subgroup $H$ of infinite index of a torsion free word hyperbolic group can be embedded in a larger quasiconvex subgroup which is the free product of $H$ and an infinite cyclic group. Since this larger quasiconvex subgroup can be chosen of infinite index we obtain in fact a method for constructing quasiconvex subgroups of word hyperbolic groups.

The result was formulated by M.Gromov in [5.3.C] of "Hyperbolic groups". The main technical statement is that a quasiconvex subgroup of a word hyperbolic group is of finite index if and only if the number of double cosets of the group modulo this subgroup is finite.

[20] G. N. Arzhantseva, *A property of subgroups of infinite index in a free group*,
Proceedings of the American Mathematical Society, **128**(11) (2000), 3205–3210, doi:10.1090/S0002-9939-00-05508-8.

Let $F$ be a finitely generated free group. It is known (by results of Greenberg, Karrass and Solitar) that if $H$ is a finitely generated subgroup of $F$ then $H$ is of infinite index if and only if there is a normal subgroup $K$ of $F$ such that $K \cap H = \{1\}$. In the present paper, we study this property of subgroups of free groups from a statistical point of view. We prove that if $H$ is a finitely generated subgroup of $F$ of infinite index, then a randomly chosen normal subgroup $K$ of $F$ has trivial intersection with $H$ with the probability tending to 1 as the lengths of the elements whose normal closure is $K$ tend to infinity. In other words, this gives a "generic property" of normal subgroups of a free group: for a fixed $H$, a generic normal subgroup of $F$ trivially intersects with $H$.

My result settles a generic variant of the (strengthened) Hanna Neumann conjecture posed in 1957. This is a statement about the rank of the intersection of two finitely generated subgroups, say $H$ and $L$, of a free group. My result gives the required upper bound on the rank of intersection $L \cap H$ for every $H$ and a "generic" $L$.

[19] G. N. Arzhantseva, *Generic properties of finitely presented groups and Howson's Theorem*,
Communications in Algebra, **26**(11) (1998), 3783-3792, doi:10.1080/00927879808826374.

The main result in this paper is a generalized version of Howson's theorem. Given $d > 0$, consider the class of finitely presented groups such that the intersection of any two subgroups with $\leq d$ generators is finitely generated. We prove that this class of groups is generic. The main step is to show that every subgroup with $\leq d$ generators is quasiconvex with respect to any finite presentation of a generic group. This is also used to prove that the class of groups with the property that every non-trivial normal subgroup with $\leq d$ generators has finite index is generic.

The techniques from [18, 19, 20] are extensively used by others, and have evolved into what is now called the *Arzhantseva-Olshanskii method*.

[18] G. N. Arzhantseva, *On the groups all of whose subgroups with fixed number of generators are free*, Fundamental and Applied Mathematics, **3**(3) (1997), 675-683 (in Russian).

We prove that in a generic finitely presented group on $m \geq 2$ generators all $\leq L$-generated subgroups of infinite index are free ($L$ is an arbitrary preassigned bound, possibly $L \gg m$) and all subgroups of finite index are not free. For proof we give a condition on defining relations which guarantees that all subgroups of infinite index with fixed number of generators are free in a finitely presented group. This condition is formulated by means of the finite labelled graphs. The result is a strong generalization of the main result of [18].

[17] G. N. Arzhantseva and A. Yu. Ol'shanskii, *Generality of the class of groups in which subgroups with a lesser number of generators are free*, Mathematical Notes, **59**(3-4) (1996), 350-355, doi:10.1007/BF02308683.

We show that in a generic $m$-generated finitely presented group every subgroup with $m-1$ generators is free. This solves question 11.75 from "Kourovka notebook" (Unsolved problems in group theory).

[16] G. N. Arzhantseva, *Generic properties of finitely presented groups,*
PhD thesis, Moscow Lomonosov State University, December 1998.

[15] G.N. Arzhantseva, A.Valette (eds.), Limits of graphs in group theory and computer science, Fundamental Sciences, EPFL Press, Lausanne, 2009, 305 pp, ISBN: 978-1-4398-0400-1.

The research articles and survey papers of this volume focus on three fields and on the interactions between them: geometric combinatorics, theoretical computer science, and geometric group theory. They highlight modern state of the art, current methods and open problems, that will be of interest both to experts and to graduate students.

[14] G. N. Arzhantseva, L. Bartholdi, J. Burillo, and E. Ventura (eds.), *Geometric group theory*, Trends in Mathematics, Birkhauser Verlag, Basel, 2007, 253 pp., ISBN: 978-3-7643-8411-1.

The volume assembles research papers in geometric and combinatorial group theory. The contributions range over a wide spectrum: limit groups, groups associated with equations, with cellular automata, their structure as metric objects, their decomposition, etc. Their common denominator is the language of group theory, used to express and solve problems ranging from geometry to logic.

[13] G. Arzhantseva, D. Kielak, T. de Laat, and D. Sawicki, *Origami expanders*, arXiv:2112.11864.

We construct the first measure-preserving affine actions with spectral gap on surfaces of arbitrary genus $g > 1$. We achieve this by finding geometric representatives of multi-twists on origami surfaces. As a major application, we construct new expanders that are coarsely distinct from the classical expanders obtained via the Laplacian as Cayley graphs of finite quotients of a group. Our methods also show that the Margulis expander, and hence the Gabber–Galil expander, is coarsely distinct from the Selberg expander.

[12] G. N. Arzhantseva, D. Osajda, *Graphical small cancellation groups with the Haagerup property*, arXiv:1404.6807.

We prove the Haagerup property (= Gromov's a-T-menability) for finitely generated groups defined by infinite presentations satisfying the graphical $C'(\bar{\lambda})$–small cancellation condition with respect to graphs endowed with a compatible wall structure. We deduce that these groups are coarsely embeddable into a Hilbert space and that the strong Baum-Connes conjecture and, hence, the Baum-Connes conjecture with arbitrary coefficients hold for them. As the main step we show that $C'(\bar{\lambda})$–complexes satisfy the linear separation property. Our result provides many new examples and a general technique to show the Haagerup property for graphical small cancellation groups.

Our method has a significant application: a construction of the first example of a group with the Haagerup property which is not $C*$-exact.

[11] G. N. Arzhantseva, C. Drutu, *Geometry of infinitely presented small cancellation groups, Rapid Decay and quasi-homomorphisms*, arXiv:1212.5280.

We prove property of Rapid Decay for all infinitely presented groups $G$ satisfying the small cancelation condition $C'(1/8)$. As a corollary, the Grothendieck metric approximation property holds for the reduced $C^*$-algebra $C_r^*(G)$ and for the Fourier algebra $A(G)$. We apply our method (standard decomposition of geodesics) to show that the kernel of the comparison map between bounded and usual group cohomology in degree 2 is infinite dimensional, with a basis of power continuum. This provides first examples of infinitely presented groups with property of Rapid Decay (with the metric approximation property, with such a large kernel of the comparison map) among direct limits of Gromov hyperbolic groups.

[10] G. N. Arzhantseva and T. Delzant, *Examples of random groups*, 2008.

We present Gromov's construction of a random group with no coarse embedding into a Hilbert space.

[9] G. N. Arzhantseva, P.-A. Cherix, *Quantifying metric approximations of discrete groups*, preprint, University of Geneva, (2008), revised version (2020), arXiv:2008.12954.

We introduce and systematically study a profile function whose asymptotic behavior quantifies the dimension or the size of a metric approximation of a finitely generated group $G$ by a family of groups $F = (G_\alpha, d_\alpha, k_\alpha, \varepsilon_\alpha)_{\alpha \in I}$, where each group $G_\alpha$ is equipped with a bi-invariant metric $d_\alpha$ and a dimension $k_\alpha$, for strictly positive real numbers $\epsilon_\alpha$ such that $\inf_\alpha \epsilon_\alpha > 0$. Through the notion of a residually amenable profile that we introduce, our approach generalizes classical isoperimetric (aka Folner) profiles of amenable groups and recently introduced functions quantifying residually finite groups. Our viewpoint is much more general and covers hyperlinear and sofic approximations as well as many other metric approximations such as weakly sofic, weakly hyperlinear, and linear sofic approximations.

[8] G. N. Arzhantseva, *An algorithm detecting Dehn presentations,* preprint, University of Geneva, 2000.

A Dehn presentation of a group $G$ leads to a known Dehn's algorithm solving the word problem for $G$. The existence of a Dehn presentation is equivalent to the word hyperbolicity of a finitely generated group. Because being word hyperbolic is a Markov property of groups there cannot exist an effective procedure for determining if a finitely presented group admits a Dehn presentation. However, there may exist an algorithm to decide whether a finite presentation of a group is a Dehn presentation. In this article we prove a result in this direction. We give an algorithm determining whether or not a finite presentation of a group is an $\alpha$-Dehn presentation for some $\frac{3}{4} \leq \alpha < 1$. Note that a Dehn presentation in the traditional sense is an $\alpha$-Dehn presentation with $\alpha = 1/2$. Observe also that any $\alpha$-Dehn presentation is a Dehn presentation.

[7] G. N. Arzhantseva, J. Díaz, J. Petit, J. Rolim, M. J. Serna, *Broadcasting on networks of sensors communicating through directional antennas,* Ambient Intelligence Computing, 1–12, Proceedings, CTI Press and Ellinika Grammata, 2003.

We propose the use of random scaled sector graphs as the basis for a model for networks of sensors communicating through radio frequency using directional antennas. We propose two broadcasting algorithms, and compare empirically their performance.

[6] G. N. Arzhantseva and J. D. P. Rolim, *Considerations for a geometric model of the web,* Approximation and Randomization Algorithms in Communication Networks, Rome, 2002, 1–11, Proceedings, Carleton Scientific.

We suggest a new geometric viewpoint on the world-wide web graph. Namely, we regard the web as a space with negative (or hyperbolic) curvature. This gives a finer information on the hyperlinked structure of the web. We also outline potential applications of this analytical approach to improve on algorithms that search and mine the web.

[5] G. N. Arzhantseva and J. D. P. Rolim, *Computability and Complexity,* e-learning theoretical course of Virtual Logic Laboratory (a project of Swiss Virtual Campus), 90 pp.

This is an extensive electronic tutorial: A complete theoretical course supported by interactive quizzes and exercises.