

Workshop on  
WORD MAPS AND STABILITY OF REPRESENTATIONS

April 11<sup>th</sup> -13<sup>th</sup> 2013  
Erwin-Schrödinger-Institute, Vienna

Organizers: Goulmira Arzhantseva | Nicolas Monod | Alain Valette

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THURSDAY, April 11

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|-------------|-------------------|---|
| 14:30-15:30 | Eugene PLOTKIN    | <i>Word maps for algebras</i>   |
| 15:45-16:45 | Liviu PAUNESCU    | <i>Linear sofic groups and algebras</i>   |
| 17:00-18:00 | Lev GLEBSKY       | <i>Almost representations of Higman's groups and cycles in repeated exponentiation mod n.</i> |
| 19:30       | CONFERENCE DINNER | <i>Gasthaus Wickerl, Porzellangasse 24a, tel. 01 317 74 89</i>                                |

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FRIDAY, April 12

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|-------------|-------------------|---|
| 09:00-10:00 | Tobias J. OSBORNE | <i>Almost implies near phenomena in quantum computation and complex quantum systems</i> |
|             | COFFEE BREAK      |   |
| 10:20-11:20 | Jakub GISMATULINE | <i>On hyperlinear and sofic groups via profinite and Bohr topology</i>                  |
| 11:30-12:30 | Andreas THOM      | <i>Ulam stability and character rigidity</i>  |
|             | LUNCH BREAK       |   |
| 14:30-15:30 | Doron PUDER       | <i>Measure preserving words are primitive</i>   |
| 15:45-16:45 | Martin KASSABOV   | <i>Images of word maps in p-adic groups</i>   |

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SATURDAY, April 13

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|-------------|-------------------|--|
| 09:00-10:00 | Kate JUSCHENKO    | <i>Small spectral radius and percolation constants on non-amenable Cayley graphs</i> |
|             | COFFEE BREAK      |  |
| 10:20-11:20 | Ori PARZANCHEVSKI | <i>Fourier expansion of word maps</i>  |
| 11:30-12:30 | Nati LINIAL       | <i>Local geometry of large combinatorial structures</i>                              |

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Eugene PLOTKIN (Bar-Ilan)

*Word maps for algebras*

In the talk a brief survey of recent results devoted to word maps on simple groups and polynomial maps on simple associative and Lie algebras will be given. Our focus is a uniform approach to word maps and parallelism/distinction between the theories. We formulate the Borel-type theorems for word maps over simple Lie algebras and associative algebras, and distinguish the cases of Engel maps modeled on Lie algebras and algebraic groups.

Liviu PAUNESCU (Bucharest)

*Linear sofic groups and algebras*

In a joint work with Goulnara Arzhantseva, we introduce linear sofic groups and linear sofic algebras. We prove that a group is linear sofic if and only if its group algebra is linear sofic. Linear soficity for groups is a priori weaker than soficity but stronger than weak soficity. We shall discuss problems in proving that linear sofic groups are sofic or that they satisfy Kaplansky's direct finiteness conjecture.

Lev GLEBSKY (Mexico)

*Almost representations of Higman's groups and cycles in repeated exponentiation mod n.*

Let  $H_{q,n} = \langle a, b, w \mid b^{-1}ab = a^q, b = w^{-1}aw, w^n = 1 \rangle$ ,  $q \in \mathbb{N}$  and  $n \in \mathbb{N} \cup \infty$ . We call  $H_{q,n}$  the Higman groups. It is due to Higman that all finite dimensional representations of  $H_{2,\infty}$  are cyclic (a and b are represented by the unit matrix). On the other hand any  $H_{q,\infty}$  has an almost representation on  $\mathbb{Z}_p = \{0, 1, \dots, p-1\}$  as follows:  $a: t \mapsto t+1 \pmod p$ ,  $b: t \mapsto qt \pmod p$  and  $w: t \mapsto q^t \pmod p$ . I am planning to discuss the problem of the estimating the number of the short cycles in the dynamical system generated by  $f(x) = q^x \pmod p$  and it's relation with the properties of  $H_{q,n}$ .

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Tobias J. OSBORNE (London)

*Almost implies near phenomena in quantum computation and complex quantum systems*

Quantum computers, if built, promise the efficient solution of many challenging computational problems using *quantum algorithms*. The natural configuration space for a quantum computer is the  $n$ -fold tensor product of  $\mathbb{C}^2$ , which represents  $n$  qubits. A quantum algorithm is then product of *quantum gates*, i.e., unitaries acting on at most 2 qubits at a time. I will describe how the quantum algorithm of quantum simulation, which provides an efficient method to simulate the dynamics of quantum spin systems, naturally exploits an "almost implies near" phenomena. In this case the dynamics is given by an *almost local automorphism* of the quasi-local algebra which is then near to a product of quantum gates. This observation has lead to a rich family of refinements, generalizations, and conjectures quantifying the interplay between almost locality and the corresponding "product locality". Connections with coarse geometry and index theory will be sketched.

Jakub GISMATULINE (Wroslaw)

*On hyperlinear and sofic groups via profinite and Bohr topology*

Glebsky and Rivera introduced the notion of a weak sofic group and used this notion in the context of profinite topology on a finitely generated free group. I will discuss connections between weak soficity and recent results of Nikolov and Segal on profinite groups. I will also present similar results about hyperlinear groups and the Bohr topology.

Andreas THOM (Leipzig)

*Ulam stability and character rigidity*

I will review some variations on Ulam stability and present some result obtained in joint work with Marc Burger and Narutaka Ozawa and recent progress on character rigidity that I have obtained in joint work with Jesse Peterson.

Doron PUDER (Jerusalem)

*Measure preserving words are primitive*

We establish new characterizations of primitive elements and free factors in free groups, which are based on the distributions they induce on finite groups. For every finite group  $G$ , a word  $w$  in the free group on  $k$  generators induces a word map from  $G^k$  to  $G$ . We say that  $w$  is measure preserving with respect to  $G$  if given uniform distribution on  $G^k$ , the image of this word map distributes uniformly on  $G$ . It is easy to see that primitive words (words which belong to some basis of the free group) are measure preserving w.r.t. all finite groups, and several authors have conjectured that the two properties are, in fact, equivalent. Here we prove this conjecture. The main ingredients of the proof include random coverings of Stallings graphs, algebraic extensions of free groups, and Mobius inversions. Our methods yield the stronger result that a subgroup of  $F_k$  is measure preserving if and only if it is a free factor. As an interesting corollary of this result we resolve a question on the profinite topology of free groups and show that the primitive elements of  $F_k$  form a closed set in this topology. Joint work with O. Parzanchevski.

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Martin KASSABOV (Cornell)

*Images of word maps in p-adic groups.*

I will discuss the images of a word maps for p-adic groups like  $G = \mathrm{SL}_n(\mathbb{Z}_p)$ . The main result is that for a fixed word  $w$  and sufficiently large  $p$  the image of  $w$  is very large and satisfies  $G = (w(G))^3$ . The exponent 3 is the best possible in general, however sometime it can be improved to 2. I will also discuss analogs of this result for discrete groups like  $\mathrm{SL}_n(\mathbb{Z})$ .

(based on joint works with N. Avni, T. Gel'nader, A. Shalev, N. Nikolov)

Kate JUSCHENKO (Nashville)

*Small spectral radius and percolation constants on non-amenable Cayley graphs*

Motivated by the Benjamini-Schramm non-unicity of percolation conjecture we study the following question. For a given finitely generated non-amenable group  $\Gamma$ , does there exist a generating set  $S$  such that the Cayley graph  $(\Gamma, S)$ , without loops and multiple edges, has non-unique percolation, i.e.,  $p_c(\Gamma, S) < p_u(\Gamma, S)$ ? We show that this is true if  $\Gamma$  contains an infinite normal subgroup  $N$  such that  $\Gamma/N$  is non-amenable. Moreover for any finitely generated group  $G$  containing  $\Gamma$  there exists a generating set  $S'$  of  $G$  such that  $p_c(G, S') < p_u(G, S')$ . In particular, this applies to free Burnside groups  $B(n, p)$  with  $n \geq 2$ ,  $p \geq 665$ . We also explore how various non-amenability numerics, such as the isoperimetric constant and the spectral radius, behave on various growing generating sets in the group. Some application of the above results to group  $C^*$ -algebras will be given. This is joint with T. Nagnibeda-Smirnova.

Ori PARZANCHEVSKI (Jerusalem)

*Fourier expansion of word maps*

Frobenius already observed that the irreducible characters of a group determine the number of times each group element is obtained as a commutator. More generally, the distribution induced by any word map has a presentation as a combination of irreducible characters, called its Fourier expansion. In this talk I will present formulas which regard the Fourier expansion of words in which some letters appear twice. These formulas give simple proofs for classical results, as well as new ones. Joint work with Gili Schul.

Nati LINIAL (Jerusalem)

*Local geometry of large combinatorial structures*

Present day technologies flood us with information, much of which takes the form of huge graphs. For various reasons (e.g., sheer size, but not only that) it is impossible to carry out any profound computations regarding these graphs. What can we do instead? One possibility is to fix a small integer  $k$  and see what  $k$ -vertex subgraphs of the graph at hand are distributed like. Two main types of questions suggest themselves: (i) Which distributions on  $k$ -vertex graphs are possible? (ii) Given this what this distribution is, what global conclusions about the graph can be drawn? In the same way many other combinatorial structures can be studied locally, e.g. permutations, partially ordered sets, tournaments, simplicial complexes and more. I will discuss some of what is known and still unknown about these questions.