REMEZ–TYPE ESTIMATES

RAFAŁ PIERZCHAŁA

Abstract

The aim of the talk is to present a Remez–type inequality for sets with cusps. Recall the classical Remez inequality: Suppose that $V \subset [0, 1]$ is measurable and $|V| > 0$. Then, for each $P \in \mathbb{R}[X]$ with $\deg P \leq n$,

$$\|P\|_{[0,\,1]} \leq T_n\left(\frac{2-|V|}{|V|}\right)\|P\|_V\,,$$

where $|V|$ denotes the Lebesgue measure of V, and T_n is the Chebyshev polynomial of degree n. There is a rich literature on the subject, including various generalizations of Remez's result. However, the available papers deal with mostly univariate or (multivariate) convex case.

The problem of Remez–type inequality in dimensions higher than one (that is, if we replace the interval $[0, 1]$ by a multidimensional set) seems to be difficult. One can expect that for convex sets it should be possible to reduce somehow the problem to dimension one. And it is the case – a version of Remez inequality for convex sets is due to Brudnyi and Ganzburg. The situation is completely different if we consider nonconvex sets – it is not even clear how to tackle the sets that have "tame" topology.