Product structures on holomorphic discrete series.

M.Pevzner (Reims)

We consider semi-simple real Lie groups G such that the associated Riemannian symmetric space G/K is a Hermitian symmetric space of tube type and the non-Riemannian one G/H is a para-hermitian symmetric space. Such symmetric spaces are usually called *causal symmetric spaces of Cayley type*. The first requirement implies that the Lie group G has holomorphic discrete series representations acting on the space of square integrable holomorphic functions on G/K. The fact that G/H is para-hermitian, i.e. has a G-invariant splitting of the tangent bundle into two isomorphic sub-bundles, allows us to build up on G/H a symbolic calculus. It turns out that one can define a G-covariant symbolic calculus on G/H generalizing the so-called convolution first symbolic calculus on $\mathbb{R}^2$.

In the present talk we discuss two different ring structures on the set of holomorphic discrete series. First one comes from the convolution of functions on the symmetric cone underlying the Hermitian symmetric space of tube type G/K.

The second one is non-commutative and is induced by the composition of operators whose symbols belong to the discrete series representations of the causal symmetric space of Cayley type G/H.

We also discuss the relationship that exists between these ring structures, intertwining operators for tensor products of holomorphic discrete series representations and the so-called Rankin-Cohen brackets.