

A Monotonicity Result for Volumes of Holomorphic Images of Balls and Eggs

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Abstract

We prove that the parametrized volume of a holomorphic image of a ball or egg strictly increases under an appropriate tensor product operation. As a Corollary we obtain a sharp upper bound for the volume of the image under a proper polynomial mapping p of degree d between balls; the bound is achieved if and only if p is homogeneous.

We express the main result in terms of operators on a Hilbert space. Let Ω be the egg domain defined by $\sum |z_j|^{2p_j} < 1$ in \mathbf{C}^n . Let $f : \Omega \rightarrow \mathbf{C}^N$ be a holomorphic mapping such that $|f|^2$ and the determinant of the complex Hessian of $\sum |f_j|^2$ are integrable. Then Df is the holomorphic mapping whose components are all the Jacobian determinants of n -tuples of the components of f . The operator M is a tensor product operation; it replaces a single component f_N of f by the n -tuple $(z_1^{p_1} f_N, \dots, z_n^{p_n} f_N)$ and leaves the others unchanged. The theorem states that the operator $M^* D^* D M - D^* D$ is nonnegative definite.