## THE $\overline{\partial}$ -NEUMANN OPERATOR AND COMMUTATORS BETWEEN THE BERGMAN PROJECTIONS AND MULTIPLICATION OPERATORS.

## FRIEDRICH HASLINGER

ABSTRACT. We prove that compactness of the canonical solution operator to  $\overline{\partial}$  restricted to (0,1)-forms with holomorphic coefficients implies compactness of the commutator  $[\mathcal{P}, \overline{M}]$  defined on the whole  $L^2_{(0,1)}(\Omega)$ , where  $\overline{M}$  is the multiplication by  $\overline{z}$  and  $\mathcal{P}$  is the orthogonal projection of  $L^2_{(0,1)}(\Omega)$  to the subspace of (0,1)-forms with holomorphic coefficients. Further we derive a formula for the  $\overline{\partial}$ -Neumann operator restricted to (0,1)-forms with holomorphic coefficients expressed by commutators between the Bergman projections and the multiplication operators by z and  $\overline{z}$ .