THE ASYMPTOTIC EXPANSION OF THE BERGMAN KERNEL ON HIGH TENSOR POWERS OF A LINE BUNDLE. II

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The purpose of this lecture is to supply more details about our approach to the asymptotic expansion of the Bergman kernel and the computation of its coefficients. In the first lecture of this series George Marinescu gave an overview and presented some applications.

If (X, ω) is a compact symplectic manifold and L is a hermitian line bundle whose curvature satisfies the pre-quantization condition $\frac{\sqrt{-1}}{2\pi} R^L = \omega$, we define the generalized Bergman kernel as the smooth kernel of the projection on the spectral space corresponding to small eigenvalues of the renormalized Bochner-Laplacian acting on $L^{\otimes p}$. If the manifold X is Kähler this corresponds to the space of holomorphic sections in $L^{\otimes p}$.

As a consequence of the Lichnerowicz formula we show that the Bochner-Laplacian has a spectral gap between small and large eigenvalues as $p \to \infty$. This permits to use the finite propagation speed for the wave equation in order to localize the problem and show the existence of the expansion. Moreover, we develop an inductive method for computing the coefficients of the expansion.

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