THE ASYMPTOTIC EXPANSION OF THE BERGMAN KERNEL ON HIGH TENSOR POWERS OF A LINE BUNDLE. I

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The asymptotic of the the Bergman kernel on high tensor powers of a line bundle has attracted a lot of attention recently. This is the first of a series of two lectures aiming to present a new approach to the existence of the asymptotic expansion and the computation of its coefficients.

Given a compact Kähler manifold X of dimension n, endowed with a positive line bundle L, we consider the space of holomorphic sections $H^0(X, L^{\otimes p})$ in the tensor powers $L^{\otimes p}$. The Bergman kernel $P_p(z, z')$ is the smooth kernel of the projection on $H^0(X, L^{\otimes p})$ and has an asymptotic expansion $P_p(z, z) = \sum_{k=0}^{\infty} b_k(z) p^{n-k}$ as $p \to \infty$.

We will discuss the asymptotic expansion of the generalized Bergman kernel in case of a symplectic manifold X. As applications, we calculate the density of states function of the Bochner-Laplacian and establish a symplectic version of the convergence of the induced Fubini-Study metric. We also explain generalizations of the asymptotic expansion for non-compact or singular manifolds and orbifolds.

The second lecture of this series by Xiaonan Ma will focus in more detail on the technical points of our approach.

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