Peter Michor

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Report of Peter W. Michor.

The aim of this project was to study actions of Lie algebras and Lie groups on manifolds. In particular, I wanted to prove the following result.

Conjecture. *Let G be a compact Lie group which acts isometrically on a complete Riemannian manifold M, such that this action admits a section: There exists a closed submanifold Σ in M which meets each orbit orthogonally.*

Then the space $\Omega_{\text{hor}}(M)^G$ of all differential forms on M which are invariant under the G-action and are horizontal in the sense that they kill each vector which is tangent to an orbit, is isomorphic to the space $\Omega(\Sigma)^{W(\Sigma)}$ of all differential forms on the section Σ, which are invariant under the action of the Weyl group $W(\Sigma)$ on Σ, where

$$W(\Sigma) = \frac{\{g \in G : g(\Sigma) \subset \Sigma\}}{\{g \in G : g|\Sigma = \text{Id}_\Sigma\}}.$$

This succeeded and the result is publish in two papers, [1] [2].

In [3] Poisson structures on Lie groups are studied in detail. First the notion of a Lie bialgebra or Lie Poisson structure is investigated, reviewed and spelled out in terms of Gauss triples, which are a generalization of Manin pairs. This is then carried over to Lie groups, in particular the double group, its various Poisson structures and dressing transformations are studied in detail. A generalization of this setting for double groups is interpreted in [4] as a generalization of the notion of an integrable system, where the usual torus action is replaced by the action of a Poisson Lie group. Paper [19] shows that by pulling back sets of functions in involution by Poisson mappings and adding Casimir functions during the process allows to construct completely integrable systems. Some examples are investigated in detail which involve the constructions of [3] and [4].

Paper [5], which seems quite spectacular to me, belongs to analysis, but it originated in questions, which arose in the study of invariants of Lie group actions. Assume that we have a polynomial whose coefficients depends smoothly on a real parameter. Can one choose the roots smoothly in that parameter? This is answered completely in the case that all roots are real: Yes, if no two roots meet of infinite order. No in general, the roots can always be chosen differentiable, but not C^1 for degree ≥ 3, in degree 2 they can always be chosen twice differentiable, but not C^2. Also complex roots are studied, and applications to perturbation theory of operators are given. In particular there is a result which is genuinely stronger than any in the book of Kato on perturbation theory of operators, which allows under some assumptions to choose the eigenvalues of a smooth 1-parameter family of selfadjoint operators with compact resolvent in a smooth way. The original question can be restated as follows: The permutation group acts by coordinate permutations on $\mathbb{R}^n$ and the elementary symmetric polynomials σ_i (the coefficients of the polynomial whose roots are the coordinates of $\mathbb{R}^n$) are generators for the

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ring of all invariants polynomials. Can one lift smooth curves in this setting over $\sigma = (\sigma_1, \dots, \sigma_h) : \mathbb{R}^n \to \mathbb{R}^n$? In [10] is is shown that the answer is yes for an arbitrary linear representation of a compact Lie group, under similar conditions as in the case of polynomials.

In the paper [11] the known upper bound $2k-1$ on the multiplicity of the k-th eigenvalue of the Laplace operator with Dirichlet boundary condition on any domain with smooth boundary in the plane is improved to $2k-3$, for $k>2$.

In the paper [13] a class of n-ary Poisson structures of constant rank is indicated. It is proved that the ternary Poisson brackets are exactly those which are defined by a decomposable 3-vector field. The key point is the proof of a lemma which tells that an n-vector ($n \geq 3$) is decomposable if and only if all its contractions with up to $n-2$ covectors are decomposable. This lemma is expanded upon in paper [18], where a new form of the Pl"ucker-relations describing thre set of all decomposable multivectors (the Grassmannian) is derived and investigated from the point of view of representation theory.

Paper [15] gives a clean derivation of the (known) result that the KdV-equation is the geodesic equation of the right invariant H^0-Riemannian metric on the Vrasoro group, and then goes on to compute the associated curvature and the equation for Jacobi fields.

Paper [14] computes the curvature tensor and the scalar curvature in the space of positive definite real matrices endowed by the Kubo-Mori inner product as a Riemannian metric.

In paper [9], on a fiber bundle without structure group the action of the gauge group (the group of all fiber respecting diffeomorphisms) on the space of (generalized) connections is shown not to admit slices.

Paper [8] treats the following subject: It is well known that the geodesic flow on the tangent bundle is the flow of a certain vector field which is called the spray $S : TM \to TTM$. It is maybe less well known that the flow lines of the vector field $\kappa_{TM} \leq TS : TTM \to TTTM$ project to Jacobi fields on TM. This could be called the 'Jacobi flow'.

In paper [7], with the help of the multigraded Nijenhuis– Richardson bracket and the multigraded Gerstenhaber bracket for every $n \geq 2$ we define n-ary associative algebras and their modules and also n-ary Lie algebras and their modules, and we give the relevant formulas for Hochschild and Chevalley cohomogy.

In paper [6], the theory of product preserving functors and Weil functors is partly extended to infinite dimensional manifolds, using the theory of C^∞-algebras.

Within the time of this project the book [G] was finished and published. Since this is a major work I enclose here the introduction.

Introduction of: The convenient setting of Global Analysis, by A. Kriegl and P. W. Michor. In the very conception of the notion of manifolds, in the Habilitationsschrift [Riemann, 1854], infinite dimensional manifolds were mentioned explicitly:

> "Es giebt indess auch Mannigfaltigkeiten, in welchen die Ortsbestimmung nicht eine endliche Zahl, sondern entweder eine unendliche Reihe oder eine stetige Mannigfaltigkeit von Grössenbestimmungen erfordert. Solche Mannigfaltigkeiten bilden z.B. die möglichen Bestimmungen einer Function für ein gegebenes Gebiet, die möglichen Gestalten einer räumlichen Figur u.s.w."

This book is meant to lay the foundations of infinite dimensional differential geometry. There are the book [Palais, 1968] and the review article [Eells, 1966] with similar titles, in which Global Analysis was based mainly on manifolds modelled on Banach space. Indeed classical calculus works quite well up to and including Banach spaces: There are existence and uniqueness for smooth ordinary differential equations (even Lipschitz), but not existence for all continuous ordinary differential equation. The inverse function theorem works well, but the theorem of constant rank offers problems, and the implicit function theorem requires splitting assumptions. There are problems with partitions of unity, with the Whitney extension theorem, and with Morse theory and transversality.

But further development has shown that Banach manifolds are not suitable for many questions of Global Analysis, by the following result, which is due to [Omori, de la Harpe, 1972], see also [Omori, 1978]: If a Banach Lie group acts effectively on a finite dimensional compact smooth manifold it must be finite dimensional itself. The study of Banach manifolds by itself is not very interesting, since they turn out to be open subsets of the modelling space for many modelling spaces, see [Eells, Elworthy, 1970].

We want to treat here manifolds which are modelled on locally convex spaces, and which are smooth, holomorphic, or real analytic in an appropriate sense. To do this we start with a careful exposition of smooth, holomorphic, and real analytic calculus in infinite dimensions. Differential calculus in infinite dimensions has already quite a long history, in fact it goes back to Bernoulli and Euler, to the beginnings of variational calculus. In this century the urge to differentiate in spaces which are more general than Banach spaces became stronger, and many different approaches and definitions were tried. The main difficulty was, that composition of (continuous) linear mappings stops to be a jointly continuous operation just at the level of Banach spaces, for any suitable topology on spaces of linear mappings. This can easily be explained in a somewhat simpler example:

Consider the evaluation $\mathrm{ev} : E \times E^* \to \mathbb{R}$, where E is a locally convex space and E^* is its dual of continuous linear functionals equipped with any locally convex topology. Let us assume that the evaluation is jointly continuous. Then there are neighborhoods $U \subset E$ and $V \subset E'$ of zero such that $\mathrm{ev}(U \times V) \subset (-1, 1)$. But then U is contained in the polar of V, so it is bounded in E, so E admits a bounded neighborhood and is thus normable.

The difficulty described here was the original motivation for the development of a whole new field within general topology, convergence spaces. But fortunately it is no longer necessary to delve into this, because in 1981, [Frölicher, 1981] and [Kriegl, 1982], [Kriegl, 1983] presented independently the solution to the question for the right differential calculus in infinite dimensions, see the monography [Frölicher, Kriegl, 1988]. The smooth calculus which we present here is the same as in this book, but our exposition is based on functional analysis rather than on category theory.

Let us try to describe the basic ideas of smooth calculus: One can say that it is a (more or less unique) consequence of taking variational calculus seriously. We start with looking at the space of smooth curves $C^\infty(\mathbb{R}, E)$ with values in a locally convex space E and note that it does not depend on the topology of E, only on the underlying system of bounded sets. This is due to the fact, that for a smooth curve difference quotients converge to the derivative much better than arbitrary converging nets or filters. Smooth curves have integrals in E if and only if a weak completeness condition is satisfied: it appeared as bornological completeness in the literature, we call it c^∞-complete. Surprisingly, this is equivalent to the condition that weakly smooth curves are smooth. All calculus in this book will be done on convenient vector spaces: These are locally convex vector spaces which are c^∞-complete; note that the locally convex topology on a convenient vector space can vary in some range, only the system of bounded set must remain the same. The next steps are then easy: a mapping between convenient vector spaces is called smooth if it maps smooth curves to smooth curves, and everything else is a theorem: existence, smoothness, and linearity of derivatives, the chain rule, and also the most important feature, cartesian closedness:

$$C^\infty(E \times F, G) \cong C^\infty(E, C^\infty(F, G)) \tag{1}$$

holds without any restriction, for a natural convenient vector space structure on $C^\infty(F, G)$: The old dream of variational calculus becomes true in a concise way. If one wants (1) and some other mild properties of calculus, then smooth calculus as described here is unique. Let us point out that on some convenient vector spaces there are smooth functions which are not continuous. This is not so horrible as it sounds, and is unavoidable if we want just the chain rule, since $\mathrm{ev} : E \times E' \to \mathbb{R}$ is always smooth but continuous only if E is normable, by the discussion above. This just tells us that the notion of topology and continuity is not the appropriate one in infinite dimensions.

An eminent mathematician once said that for infinite dimensional calculus each serious application needs its own foundation. By a serious application one obviously means some application of a hard inverse function theorem. These theorems can be proved, if by assuming enough a priori estimates one creates enough Banach space situation for some modified iteration procedure to converge. Many authors try to build their platonic idea of an a priori estimate into their differential calculus. We think that this makes the calculus inapplicable and hides the origin of the a priori estimates. We believe that the calculus itself should be as easy to use as possible, and that all further assumptions (which most often come from ellipticity of some nonlinear partial differential equation of geometric origin) should be treated separately, in a setting depending on the specific problem. We are sure that in this sense the setting presented here (and the setting in [Frölicher, Kriegl, 1988]) is universally usable for most applications. To give a basis to this statement we present also the hard implicit function theorem of Nash and Moser, in the approach of [Hamilton, 1982] adapted to convenient calculus, but we give none of its serious applications.

A surprising and very satisfying feature of the notion of convenient vector spaces is that it is also the right setting for holomorphic calculus as shown in [Kriegl, Nel, 1985], for real analytic calculus as shown by [Kriegl, Michor, 1990], and also for multilinear algebra. We give an extensive treatment of all these topics.

The middle part of the book is then devoted to the theory of infinite dimensional manifolds and Lie groups and some of its applications. We treat here only manifolds described by charts although this limits cartesian closedness of the category of manifolds drastically, see 44.14 and section 17 for more thorough discussions. We start by treating existence of smooth bump functions and smooth partitions of unity. Then we investigate tangent vectors seen as derivations or kinematically (via curves): these concepts differ, and there are some surprises even on Hilbert spaces, see 30.4. Accordingly, we have different kinds of tangent bundles, vector fields, differential forms, which we list in a somewhat systematic manner. The theorem of De Rham is proved, and a (small) version of the Frölicher-Nijenhuis bracket in infinite dimensions is treated. Finally, we discuss Weil functors (certain product preserving functors of manifolds) as generalized tangent bundles. Infinite dimensional Lie groups can be pushed surprisingly far: Exponential mappings are unique if they exist. A stronger requirement (leading to regular Lie groups) is that one assumes that smooth curves in the Lie algebra integrate to smooth curves in the group in a smooth way (an 'evolution operator' exists). This is due to [Milnor, 1984] who weakened the concept of [Omori, Maeda, Yoshioka, 1982]. It turns out that regular Lie groups have strong permanence properties. Up to now (April 1997) no non-regular Lie group is known. Connections on smooth principal bundles with a regular Lie group as structure group have parallel transport, and for flat connections the horizontal distribution is integrable. So some (equivariant) partial differential equations in infinite dimensions are very well behaved, although in general there are counter examples in every possible direction.

The rest of the book describes applications: manifolds of mappings between finite dimensional manifolds, diffeomorphism groups, their classifying spaces, direct limit manifolds, and a concise and easy approach to smooth vectors and smoothness of infinite dimensional (unitary) representations.

The work on this book was done from 1989 onwards, the material was presented in our joint seminar and elsewhere several times, which led to a lot of improvement. We want to thank all participants, who devoted a lot of attention and energy, in particular our (former) students who presented talks on that subject, also those who helped with proofreading or gave good advise: Eva Adam, Andreas Cap, Stefan Haller, Ann and Bertram Kostant, Grigori Litvinov, Josef Mattes, Martin Neuwirther, Tudor Ratiu, Konstanze Rietsch, Hermann Schichl, Erhard Siegl, Josef Teichmann, Klaus Wegenkittl.

Beitrag von Stefan Haller.

Ein Teil meiner Arbeit war es Einfachheit bzw. Perfektheit gewisser Diffeomorphismengruppen zu zeigen. Das Problem wird in zwei Schritten angegangen: Zuerst wird es für eine spezielle Mannigfaltigkeit jeder Dimension gelöst (Torus) und dann auf beliebige Mannigfaltikeiten übertragen. Der zweite Schritt wurde bisher in ein Fragmentationslemma und ein Deformationslemma zerlegt. Für modulare Diffeomorphismengruppen ist es mir gelungen diese beiden Konstruktionen durch eine einzige zu ersetzen. Die ursprünglichen beiden Schritte erscheinen nun als Teile eines Homotopieoperators zwischen gewissen Kettenkomplexen.

Im Weiteren habe ich mich mit sogenannten lokal konform symplektischen (l.k.s.) Mannigfaltigkeiten auseinanergesetzt. Diese bilden eine Erweiterung der Klasse der symplektischen Mannigfaltigkeiten. Diese geometrische Struktur ist deshalb interessant, weil dies genau die gerade-dimensionalen Blaetter von Jacobi-Mannigfaltigkeiten sind (die ungerade-dimensionalen sind genau die Kontakten-Mannigfaltigkeiten). Ich konnte zeigen, daß die Automorphismengruppe einer l.k.s. Mannigfaltigkeit eine unendlichdimensionale Lie Gruppe ist. Ich habe dann die beiden Invarianten die es für die symplektische Diffeomorphismgruppe gibt (Flux, Calabi) auf l.k.s. Mannigfaltigkeiten erweitert. Außerdem erscheint im l.k.s. Fall eine weitere Invariante, die für symplektische Mannigfaltigkeiten immer verschwindet. Ich konnte zeigen, daß der Kern der Calabi Invariante einfach und perfekt ist. Dies ist eine Verallgemeinerung eines bekannten Theorems von A. Banyaga für symplektische Mannigfaltigkeiten. Weiters habe ich die Kommutatorreihe der Automorphismengruppe einer l.k.s. Mannigfaltigkeit berechnet. Es zeigte sich, daß diese fast immer gerade aus den Kernen der obigen Invarianten besteht.

Auch konnte ich zeigen, daß die Automorphismengruppe einer l.k.s. Mannigfaltigkeit (als abstrakte Gruppe) sowohl die zugrundeliegende Mannigfaltigkeit als auch die l.k.s. Struktur bestimmt. Gleiches gilt auch für die Kerne der verschiedenen Invarianten. Ähnliche Resultate gibt es für eine Reihe von geometrischen Strukturen (Erlanger Programm).

Beitrag von Josef Teichmann.

Im Rahmen meiner Dissertation bei Peter W. Michor beschäftige ich mich mit einer Verbesserung und Fundierung der Theorie zur Lösung von Differentialgleichungen auf unendlichdimensionalen Liegruppen. Bisher wurde dieses Problem durch eine bisweilen komplizierte Rückführung auf Problemstellungen auf Banachräumen gelöst. Die zum Teil bemerkenswerten topologischen und metrischen Eigenschaften von unendlichedimensionalen Liegruppen wurden nicht weiter untersucht.

Ein erster in der Dissertation verfolgter Ansatz gestattet es mit topologisch-metrischen Bedingungen an die (unendlichdimensionale) Liegruppe die Existenz von Lösungen invarianter (nicht) autonomer Differentialgleichungen zu charakterisieren. Außerdem wurde bewiesen, daß die Existenz von Lösungen genannter Differentialgleichungen äquivalent ist zur Existenz aller möglichen Produktintegral-Approximationen, zumindest auf einer alle bekannten Liegruppen umfassenden Kategorie von Liegruppen. Dieses Resultat wird in der Arbeit [27] veröffentlicht werden.

Als weiterer Ansatz wurde eine Theorie zur Lösung nicht autonomer linearer Differentialgleichungen auf geeigneten Vektorräumen entwickelt, die auch für sich interessant ist: Sie beinhaltet eine geeignete Hille-Yosida Theorie und eine Approximationstheorie mit Produktintegralen. Im Gegensatz zu bisher publizierten Theorien wird auf die Möglichkeit hingewiesen und auch Nutzen daraus gezogen, daß es genügt, glatte Halbgruppen zu untersuchen. Außerdem wurde erstmals eine anwendbare Approximationstheorie entwickelt. Die geeignete Hille-Yosida Theorie samt einigen Anwendungen wird in [26] veröffentlicht werden, wobei die angebotenen Lösungen wesentlich einfacher sind als die bisher bekannten. Mit Hilfe der Theorie zu Produktintegralen auf geeigneten Räumen gelingt es auch, an das glatte Produkt auf Liegruppen Bedingungen zu formulieren, die es gestatten auf Regularität zu schließen. Resultate dazu werden in [25] publiziert werden.

Die Beobachtung, daß die adjungierte Darstellung $Ad : G \to \mathfrak{g}$ in vielen Fällen initial ist, führt in ein weiteres Feld von Anwendungen linearer Theorien auf die Lösbarkeit von Differentialgleichungen auf Liegruppen. Dieser dritte Ansatz stellte sich aber als nur teilweise fruchtbar heraus. Allerdings sind die abgeleiteten notwendigen Bedingungen via dem geeigneten Hille-Yosida Theorem interessant.

Auf der Suche nach interessanten Gegenbeispielen gelang es zusammen mit Peter W. Michor eine Lücke in der Theorie regulärer abelscher Liegruppen zu füllen: Reguläre abelsche Liegruppen sind mittlerweile genau als Quotienten mit 'diskreten' Untergruppen von geeignete Räumen erkannt (siehe [12]).

References

[G] Kriegl, Andreas; Michor, Peter W., *The Convenient Setting for Global Analy-*

sis, Surveys and Monographs, Vol. 53, AMS, Providence, 1997.

[1] Michor, Peter W., *Basic differential forms for actions of Lie groups*, Proc. AMS **124, 5** (1996), 1633–1642.

[2] Michor, Peter W., *Basic differential forms for actions of Lie groups II*, Proc. AMS **125** (1997), 2175-2177.

[3] Alekseevsky, Dmitry; Grabowksi, Janusz; Marmo, Giuseppe; Michor, Peter W., *Poisson structures on double Lie groups*, J. Geom. Physics **26** (1998), 340-379.

[4] Alekseevsky, Dmitry; Grabowksi, Janusz; Marmo, Giuseppe; Michor, Peter W., *Completely integrable systems: a generalization*, Modern Physics A **12** (1997), 1637–1648.

[5] Alekseevky, Dmitri; Kriegl, Andreas; Losik, Mark; Michor; Peter W., *Choosing roots of polynomials smoothly*, Israel J. Math. **105** (1998), 203-233, ESI Preprint 314. math.CA/9801026.

[6] Kriegl, Andreas; Michor; Peter W., *Product preserving functors of infinite dimensional manifolds*, Archivum Mathematicum (Brno) **32,4** (1996), 289–306.

[7] Michor, Peter W.; Vinogradov, Alexandre, *n-ary Lie and Associative Algebras*, Rend. Sem. Mat. Univ. Pol. Torino **53** (1996), 373-392.

[8] Michor, Peter W., *The Jacobi Flow*, Rend. Sem. Mat. Univ. Pol. Torino **54** (1996), 365-372.

[9] Michor, P.W.; Schichl, H., *No slices on the space of generalized connections*, ESI Preprint 453, Acta Math. Univ. Comenianiae **66** (1997), 221–228.

[20] Michor, Peter W.; Ratiu, Tudor, *Curvature of the Virasoro-Bott group*, J. Lie Theory **8** (1998), 293-309.

[10] Alekseevky, Dmitri; Kriegl, Andreas; Losik, Mark; Michor; Peter W., *Lifting smooth curves over invariants for representations of compact Lie groups*, submitted, Transformation groups.

[11] Thomas Hoffman-Ostenhof, Peter W. Michor, Nikolai Nadirashvili, *Bounds on the multiplicity of eigenvalues for fixed membranes*, to appear, Geometric And Functional Analysis, ESI Preprint 514. math.AP/9801090.

[12] Peter W. Michor and Josef Teichmann, *Description of infinite dimensional abelian regular Lie groups*, to appear, J. Lie Theory..

[13] Michor, Peter W.; Vaisman, Izu, *A note on n-ary Poisson brackets*, to appear, Rend. Circ. Matem. Palermo, Suppl., ESI Preprint 663. math.SG/9901117.

[14] Michor; Peter W.; Petz, Dénes, *On the curvature of a certain Riemannian space of matrices*, in preparation.

[15] Michor, Peter W.; Misiolek, Gerard; Ratiu, Tudor, *Geodesics on some infinite dimensional Lie groups with a view to completely integrable systems*, in preparation.

[16] Alekseevky, Dmitri; Kriegl, Andreas; Losik, Mark; Michor; Peter W., *The Riemannian geometry of orbit spaces*, in preparation.

[17] Dmitri Alekseevsky, Peter W. Michor, Wolfgang Ruppert, *Extensions of Lie algebras and Lie groups*, in preparation.

[18] Mike Eastwood, Peter W. Michor, *Some remarks on the Plücker relations*, to appear, Rend. Circ. Matem. Palermo, Suppl..

[19] Janusz Grabowski, Giuseppe Marmo, Peter W. Michor, *Construction of completely integrable systems by Poisson mappings*, to appear, Modern Physics Letters A.

[21] Haller, Stefan, *On perfectness and simplicity of certain diffeomorphism groups*, Dr. rer. nat Dissertation, Universität Wien, 1998.

[22] Haller, Stefan; Rybicki, Tomasz, *On the group of diffeomorphisms preserving a locally conformal symplectic structure*, Ann Global Anal. Geom. **17** (1999), 475–502.

[23] Haller, Stefan; Rybicki, Tomasz, *On the perfectness of nontransitive groups of diffeomorphisms*, submitted, Fundam. Math..

[24] Teichmann, Josef, Dr. rer. nat Dissertation, Universität Wien, 1998.

[25] Teichmann, Josef, *A convenient approach to Trotter's formula*, submitted, Semigroup Forum.

[26] Teichmann, Josef, *Convenient Hille-Yosida theory*, submitted, J. Funct. Anal..

[27] Teichmann, Josef, *Regularity of infinite-dimensional Lie groups by metric space methods*, submitted, Tokyo J. Math..

Vorträge 1996-1999 des Projektes P10037-PHY

P. Michor: Choosing roots of polynomials smoothly, 16th Winter school on Geometry and Physics, Srni, 13.–20.1.1996

P. Michor: Wurzeln von Polynomen differenzierbar wählen, Kolloquium, Institut für Mathematik, Universität Wien, 31.1.1996

P. Michor: Weil functors in infinite dimensions, Central European Seminar on Differential Geometry, Brno, 17.5.1996

P. Michor: Basic differential forms for actions of compact Lie groups, Seminar, ESI, 22.5.1996

P. Michor: Choosing roots of polynomials differentiably - alias - lifting smooth curves over invariants, Seminar, ESI, 26.6.1996

P. Michor: Basic differential forms for actions of compact Lie groups, Seminar Sophus Lie, Universität für Bodenkultur, Wien, 12.7.1996

P. Michor: Choosing roots of polynomials differentiably - alias - lifting smooth curves over invariants, Conference on Differential Geometry (Satellite Conference), Budapest, 27.7.-30.7.1996

P. Michor: Cutting a hole into the projective plane give you the Möbius strip. Abschluss-Vortrag, 2nd Junior Mathematical Congress, Miskolc, Ungarn, 29.7.1996-2.8.1996.

P. Michor: $\mathfrak{g}$-manifolds as generalizations of principal bundles, invited lecture, Conference differential Geometry and Physics, IIASS, Vietri sul'mare, 30.9.1996-4.10.1996.

P. Michor: Choosing roots of polynomials smoothly and perturbation of operators on Hilbert space, 17th Winter school on Geometry and Physics, Srni, 12.–19.1.1997

P. Michor: Choosing roots of polynomials smoothly and perturbation of operators on Hilbert space, Berufungsvortrag, Universität Linz, 3.3.1997

P. Michor: Choosing roots of polynomials smoothly and perturbation of operators on Hilbert space, All institute seminar, Institute of Mathmatics, Hungarian Academy of Sciences, 26.5.1997

P. Michor: Regular infinite dimensional Lie groups, main lecture, Conference on infinite dimensional Lie groups, Banach Center, Warsaw, 8.-12.9.1997

P. Michor: Wurzeln von Polynomen glatt wählen und Störungstheorie von Operatoren auf Hilberträumen, ÖMG-Kongress-Salzburg, Sektion Analysis, 22.-26.9.1997

P. Michor: Choosing roots of polynomials smoothly and perturbation of operators on Hilbert space, Seminar, Centre de Mathematiques, Luminy, Marseille, 21.10.1997.

P. Michor: Bounds on the multiplicity of eigenvalues for the Laplace operator on fixed membranes, 18th Winter school on Geometry and Physics, Srni, 10.–17.1.1998

P. Michor: Bounds on the multiplicity of eigenvalues for the Laplace operator on fixed membranes, 18th Winter school on Geometry and Physics, Srni, 10.–17.1.1998

P. Michor: Invariante Differentialformen fuer Wirkungen von Lie Gruppen Berufungsvortrag, Graz, 9.5.1998

P. Michor: A review on infinite dimensional Lie groups Conference 'Transformation groups in differential geometry' Levico Terme, Trento, I, 25.–29.5.1998

P. Michor: Extensions of groups, Lie algebras, and Lie groups. Conference 'Differential Geometry and its Applications' Brno, CZ, 10-14.8.1998

P. Michor: Regular infinite dimensional Lie groups. Conference if the Italian research project 'Singularity, Integrability, and Symmetry' on 'Group of Differomorphisms and Representation Theory of infinite dimensional algebras' in Villa Gualino, Torino, October 1-3, 1998.

P. Michor: The Korteweg-De Vries equation as geodesic equation on Diffeomorphism groups. Same conference as

P. Michor: Erweiterungen von Gruppen und Lie-Algebren. Kolloquium des Mathematischen Institutes, Universität Wien, October 25, 1998.

P. Michor: n-ary Poisson structures 19th Winter school on Geometry and Physics, Srni, 9.–16.1.1999

P. Michor: Geodesics on infinite dimensional Lie groups with a view towards completely integrable systems. Dynamical Systems Seminar, ESI, 22.3.1999.

P. Michor: Geodesics on infinite dimensional Lie groups with a view towards completely integrable systems. Conference: integrable systems, Vietry/Salerno/I, 14.6.-18.6.1999

P. Michor: Smooth perturbation theory of unbounded operators. Conference: Geometric spectral theory and Schrödinger Operators, Matrei/A, 5.7.-10.7.1999

St. Haller: Second Calabi invariant for the group of Poisson diffeomorphisms. Workshop on Lie groups and Lie algebras in infinite dimensions Warsaw, September 8-13, 1997.

St. Haller: Fragmentation and deformation for modular groups of diffeomorphisms. 18th Winter School in Geometry and Physics Srni, January 10-17, 1998.

J. Teichmann: Convenient Hille-Yosida-Theory, 19th Winter school on Geometry and Physics, Srni, 10-17.01.1999

J. Teichmann: Geeignete Hille-Yosida-Trotter Theorie, Mathematisches Kolloquium, Universität Wien, Institut für Mathematik, 17.03.1999

J. Teichmann: Lipschitz-metrics on infinite dimensional Lie groups, Conference on Geoemtry and Topology of Manifolds, Krynica Górska, 24-29.04.1999