

Leray in Edelbach

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Does your hometown have any mathematical tourist attractions such as statues, plaques, graves, the café where the famous conjecture was made, the desk where the famous initials are scratched, birthplaces, houses, or memorials? Have you encountered a mathematical sight on your travels? If so, we invite you to submit to this column a picture, a description of its mathematical significance, and either a map or directions so that others may follow in your tracks.

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This is a most unlikely place for the mathematical tourist to visit. In fact, it is off-limits for tourists of any kind. Photographing, filming, even drawing, is prohibited by law, as signposts tell you sternly, and trespassers will be punished. If they survive at all, that is. Indeed, the signposts also warn you of *LEBENSGEFAHR*, meaning mortal danger. You are in a military zone, and had better watch out. Don't step on any mines, and avoid getting shot, says an urgent inner voice.

But this is ridiculous. We are in Austria, after all, with almost sixty years of peace and prosperity behind us. Nobody wants any trouble. Let's not get caught, that's all.

Welcome to Edelbach, or what is left of it. The place is not easily found on a map: it ceased to exist many years ago, during the darkest days of Austrian history. Nobody lives here any longer. The main road between Vienna and Prague is a couple of miles to the north, but it can be neither seen nor heard. An eerie silence hangs over the place. All that remains of the former village are a few stone-heaps between thickets of fir trees, and a small, abandoned graveyard. To the north of it, a modern fence surrounds a vast ammunition depot. It is very well guarded, and you can be sure, by now, that binoculars are fixed on you.

This place was once a camp for prisoners of war, mostly French officers. An "Offizierslager"—or Oflag for short: the bureaucrats of the Third Reich were fond of abbreviations. Oflag XVIIA was the birthplace of a substantial part of algebraic topology. Spectral sequences and the theory of sheaves were fathered here by an artillery lieutenant named Jean Leray, during an internment lasting from July 1940 to May 1945 ([Sch 1990][Eke 1999][Gaz 2000]).

In the annals of science one finds several examples of first-rate mathematical research conducted by prisoners of war. The Austrian Eduard Helly, for instance, wrote a seminal paper

on functional analysis in the Siberian camp of Nikolsk-Ussurisk, during World War I; and a century before, the Napoleonic officer Jean-Victor Poncelet developed projective geometry while in Russian captivity for five years. This may sound as if the monastic reclusion and monotonic regularity of confined life provided ideal conditions for concentrating the mind. And indeed, André Weil wrote that "nothing is more favourable than prison for the abstract sciences" [Weil 1991]. He wrote this while he was in prison, and managed, during his months of captivity, to find some of his major theorems. But he had a prison cell for himself, could receive visits from his family, and knew assuredly, to use his words, "captivity from its most benign side only." The physical and psychic deprivations of years in a POW camp, with its overcrowding, sickness, hunger, and biting cold, on top of the boredom and uncertainty, were something else: in these conditions, intense intellectual pursuit must have been a desperate means for keeping hold of sanity.

The prisoners of Edelbach founded a "University in Captivity." Of the 5,000 inmates of the camp, of which a few hundred were Polish and the rest French, almost 500 got degrees, and their diplomas were all officially confirmed in France after the war. The fact that Jean Leray had been the director, or *recteur*, of this impromptu university must have helped with the French authorities. His academic credentials were impressive: he had received his doctorate at the elite *École Normale Supérieure* in Paris, and had been professor at the Université de Nancy before being drafted into the war. His joint work with the Polish mathematician Juliusz Schauder (later a victim of the Holocaust) developed a topological invariant to prove the existence of solutions of partial-differential equations. This earned him in 1940 the *Grand Prix* in mathematics from the *Académie des Sciences de Paris*.


MILITÄRISCHES SPERRGEBIET!
Lebensgefahr!
Betreten und Befahren,
Fotografieren, Filmen und Zeichnen,
gesetzlich verboten und strafbar!



Fig. 1. Tourists are not exactly welcome in Edelbach nowadays, but what can you expect from an ammunition depot?

But Leray was not the only distinguished scientist in the Oflag. There was the embryologist Étienne Wolff, by all testimonies a driving force behind the university, but obliged, for racial reasons, to keep discreetly in the background. Étienne Wolff later became professor at the *Collège de France*, and member of the *Académie des Sciences de Paris* as well as of the *Académie Française*. Another luminary was François Ellenberger, a future president of the *Société Géologique de France*. The geologists at Oflag XVII had to content themselves with the stones they could find in the prison yard. Their laboratory was an old kitchen which they could use for a few hours daily.

Eventually, friends and relatives from France were permitted to send books. Over the years, Leray received a small library from his former teacher Henri Villat [Sch 1990], [Ell 1948].

From eight in the morning to eight in the evening, Barrack 19 housed lectures on law and biology, on psychology and Arab language, on music and moral theology, on horse-raising (by a Polish fellow-officer, *bien sûr!*), on

public finances, and on astronomy. The course on probability was given by Lieutenant Jean Ville, who had published, just before the war, an ingenious elementary proof of von Neumann's minimax theorem [Poll 1989].

Recteur Leray lectured mostly on calculus and topology. He had succeeded in hiding from the Germans the fact that he was a leading expert in fluid dynamics and mechanics (a *mécanicien*, as he liked to say). He turned, instead, to algebraic topology, a field which he deemed unlikely to spawn war-like applications. This led, first, to some notes in the *Comptes Rendus de l'Académie des Sciences de Paris*, and eventually to a three-part work "Algebraic topology taught in captivity," which was submitted in 1944 to the *Journal des Mathématiques Pures et Appliquées*, through the good offices of Heinz Hopf from neutral Switzerland, who endorsed it enthusiastically. It was published, after Leray's release, in 1945 [CRAS 1942] [JMPA 1945].

The university's curriculum shows that on Sunday nights, the prisoners could listen to a lecture giving "practical advice for constructing an inex-

pensive house," before having to return to their cheerless cold quarters. The barracks consisted of two rooms housing 100 inmates each, one small kitchen, and one toilet with eight wash-basins. There was a special building for the showers: each officer could use it twice a month. Half of one barrack was used as a chapel. More than seventy of the prisoners were priests, and each could say mass daily if he wished. The captives founded a first-rate choir and a theatre group, and soon set up their own sports stadium, named *stade Pétaïin*. The prisoners even managed to produce, behind the back of their guards, a documentary film of about thirty minutes' length, entitled *Sous le Manteau* ("Beneath the Cloak," because the camera had always to remain hidden). Three versions of it have survived to this day ([1989][Kus 2004]).

As in many other POW camps, the captives printed their own newspaper, a weekly called *Le Canard . . . en KG*. KG is Wehrmacht shortspeak meaning *Kriegsgefangener*, or prisoner of war, and the French would pronounce it as *Le canard encagé* (The caged duck), a pun referring to the celebrated *Le Ca-*

nard Enchaîné (The duck in chains), which was, and still is, a hugely popular satirical journal in France. The prisoners' version was not permitted to comment on politics, satirically or otherwise: it was filled with harmless caricatures, theatre bills, sports news, crossword puzzles, and announcements of special lectures. Nothing about the war, or about the conflicts dividing the French community into what, with hindsight, was simply the issue of *collaboration* vs. *résistance*, but seemed much more confusing at the time. The Vichy régime tried to foster a network of "*hommes de confiance*," but an underground *résistance* group, who called themselves the mafia, eventually became the dominating force in the camp. For many of the prisoners, the dilemma was whether to become a civilian worker in Germany, with a freedom . . . of sorts, or to stick it out behind the barbed wire, in the hope that the legal status of a captive officer would protect them from the worst. For Leray, who in 1933 had witnessed in Berlin the accession of Hitler to power, collaboration was never an issue.

When Leray later spoke about Edelbach, he located it "near Austerlitz, in Austria" [Sch 1990]. Actually, Austerlitz is across the border, in Czechia, and

not really nearby (some 83 kilometres away). Edelbach is closer to Vienna than to Austerlitz, but for the defeated French officers, the thought of being near the site of the great Napoleonic victory—"à portée de canon d'Austerlitz," as some liked to say—must have been a solace. At first, they all had hoped to be back in France by the end of 1940. The war seemed over. When this proved an illusion, many fell prey to depression and to homesickness. Leray and his academic colleagues used to meet every evening in the highest, southernmost corner of the camp, and watch, weather permitting, the sunset over "la petite France."

Needless to say, the French did not merely bemoan their fate. Some tried to change it. The prison guards became experts at discovering tunnel entrances beneath the barracks. They were so good at it that they overlooked a tunnel entrance which was out in the open, right under their noses. It was through this 90-meters-long tunnel that on the nights of September 17 and 18, 1943, no fewer than 132 prisoners decamped. It was the greatest escape from a POW camp in World War II, and its story is almost unknown [Kus 2004].

The prisoners had established an open-air theatre, called *Théâtre de la Verdure*. They were allowed to deco-

rate it with twigs and greenery, hiding it partially from the guard towers. Because delegates of the International Red Cross had found that the camp lacked protection against Allied air raids, the POWs were told to dig a few trenches, and were even provided with shovels and wheelbarrows. Under a plank bridging one of the dug-outs, they started burrowing in earnest. The tunnel grew quickly, by almost a metre per day, although water kept flooding in. After some time, ventilation became a problem: through a hose made from tin cans, fresh air had to be pumped into the gallery, which was less than two feet wide and three feet high. In parallel, a tailor shop produced civilian clothes, and the printing press prepared maps and forged documents. Canned food was hoarded in hidden depots.

The first group left on a Saturday night. Their escape went unnoticed during Sunday, because some of the guards were on holiday. The second group left on the following night. Most of the runaways hoped to pass for French civilians, of whom there were many working in Germany at that time. The first escapees were arrested and returned to the camp by the police even before the break-out was discovered by the military guards. Eventually,



Fig. 2. Lieutenant Jean Leray, POW, became the rector of the "University in Captivity." The picture on the right shows him with his Edelbach colleagues. Some would later join him at the Sorbonne or the Collège de France [Gaz 2000].

UNIVERSITÉ OFLAG XVII A

PROGRAMME DES COURS & CONFÉRENCES

Cours d'Allemand		Cours de Français		Cours de Mathématiques		Cours de Sciences		Cours de Lettres	
Allemand 1 ^{er} degré	E 1	Grammaire de français	E 1	Arithmétique	E 1	Physique	E 1	Philosophie	E 1
Allemand 2 ^e degré	E 2	Grammaire de français	E 2	Algèbre	E 2	Chimie	E 2	Philosophie	E 2
Allemand licence	E 1	Grammaire de français	E 1	Calcul différentiel	E 1	Biologie	E 1	Philosophie	E 1
Droit civil 2 ^e année	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2
Le Cervin (rép.)	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Espagnol 2 ^e degré	E 1	Grammaire de français	E 1	Calcul différentiel	E 1	Biologie	E 1	Philosophie	E 1
Cours de probabilités	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2
Vinances publiques	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Histori. de la messe	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Cercle d'astronomie	E 1	Grammaire de français	E 1	Calcul différentiel	E 1	Biologie	E 1	Philosophie	E 1
Latin élém.	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2
Quotidien des investit.	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Prép. brevet agric.	E 1	Grammaire de français	E 1	Calcul différentiel	E 1	Biologie	E 1	Philosophie	E 1
Espagnol 1 ^{er} degré	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2
Crédit bancaire	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Baptême et 1 ^{re} Comm.	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Syndicat chrétien	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Horticulture d'ornem.	E 1	Grammaire de français	E 1	Calcul différentiel	E 1	Biologie	E 1	Philosophie	E 1
Calcul différ. (licence)	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2
Littérature du XX ^e s.	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Maison Déco. et amén. int.	E 1	Grammaire de français	E 1	Calcul différentiel	E 1	Biologie	E 1	Philosophie	E 1
Droit civil 1 ^{re} année	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2
Vie hors de l'organisme	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Chine 1 ^{re} vue d'ensemble	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Exploit. et entret. du bois	E 1	Grammaire de français	E 1	Calcul différentiel	E 1	Biologie	E 1	Philosophie	E 1
Calcul différ. (licence)	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2
Enfance coupable	O	Grammaire de français	O	Calcul différentiel	O	Biologie	O	Philosophie	O
Educ. civique école prim.	E 2	Grammaire de français	E 2	Calcul différentiel	E 2	Biologie	E 2	Philosophie	E 2

Fig. 3. The curriculum of the Université en Captivité [Poll 1989]. As Leray later said, “students had no other distraction than their studies. They had little to eat, and little to keep warm; but they were courageous.” [Sch 1990]

only two fugitives managed to reach France.

Soon after, a panel of agitated German officers, including several generals, visited the Oflag, where they were filmed surreptitiously by the French prisoners. The commission decided to play down the escape—it did not show the Wehrmacht in a favourable light. The prisoners were sternly told that they should not try it again. Handbills were distributed warning that “breaking out is no longer a sport” and that death zones were waiting for the runaways. Half a year later, 76 British flyers escaped from Oflag III Luft in Sagan. This time, the Wehrmacht could no longer keep it a secret from Hitler and Himmler. Only three of the fugitives reached England; 50 were shot.

During the five years that Leray spent in Oflag XVII, battles raged from one end of Europe to the other, at no time touching Edelbach. Nevertheless, the booming of great guns and the angry buzz of Stukas could be heard at all times by the inmates of the camp. Indeed, Oflag XVII was located within an evacuated zone, strictly off-limits for civilians, the *Truppenübungsplatz*

Döllersheim. This was the largest military training ground in central Europe, twenty kilometres in diameter, larger than the dukedom of Liechtenstein. A few months after the *Anschluss*, Hitler’s annexation of Austria in 1938, the German army had taken over the ground. Forty-five villages with more than seven thousand inhabitants were hastily evacuated, and huge mechanised forces rattled across the fields, taking little notice of the fact that the harvest was not yet in. The Wehrmacht had to live up to its new and as yet untested doctrine of the *Blitzkrieg*. The barracks which Leray and his fellow-prisoners were soon to use were erected originally to house the first German soldiers claiming the exercise grounds. Very soon, the *Truppenübungsplatz* proved an ideal stepping-stone for the armies which were assembling to invade and dismember nearby Czechoslovakia, in spring 1939, and for preparing the assault on Poland during the following summer months [Poll 1989].

The fact that both the father and the mother of Adolf Hitler had been born in the region, which was so suddenly and ruthlessly evacuated, gave rise to spec-

ulations. One of the closest associates of the Führer, Hans Frank, would later write, in his death cell in Nuremberg, that Hitler intended thereby to erase all traces of his origins [Frank 1953]. He reported that these traces could reveal a dark secret, the shame and scandal of the Third Reich: namely, that Hitler had a Jewish grandfather. This rumour, which had been widespread in Nazi Germany and still finds adherents today, has been debunked by scores of historians since. Hitler’s father had been born out of wedlock, as Alois Schicklgruber, and was later to change his name, but the *Führer* was far too powerful to have felt threatened by slurs concerning his ancestry. In fact, when the villages around Döllersheim were evacuated, all church archives were properly stored. They are preserved to this day. For years, the Wehrmacht had been looking for a king-sized training ground to accommodate its frantic growth, and to manoeuvre with its new weapons, whose range would not fit into existing exercise areas. The Waldviertel (or woods district), with its poor soil and its sparse, lowly population was perfectly suited: a hilly

plateau, some 600 metres above sea level, with long, bitterly cold winters, and no reputation for hospitality.

It is clear that Hitler had no emotional ties to the Waldviertel. The propaganda from the Goebbels ministry had hailed it as the *Ahnengau*, the cradle of the ancestors, and the humble dwellers of the tiny hamlet of Grosspoppen, led by their inn-keeper, had conferred honorary citizenship on Hitler in 1932, when he was a rising young politician and demagogue. In return, they first got scolded by the authorities of Lower Austria (who pointed out that the action was legally void because Hitler was no longer an Austrian citizen), then frowned upon by the Viennese regime, which was engaged in a hopeless struggle against illegal Nazis, and finally, right after the revels of the annexation, expelled from their land without further ado. No account was taken of the fact that 220 out of the 220 citizens of Grosspoppen had voted for the *Anschluss*. In fact their hamlet, which obstructed a planned artillery range, was the first to become *menschenein* (the callous Nazi expression for “evacuated”) and be knocked down.

A fortunate few were compensated with hastily built ersatz farms, not too far away. Others were given provi-

sional quarters and the promise of a settlement after the war. In 1942, all evacuees were offered a special reduction on a richly produced coffee-table book, *Die alte Heimat*, complete with pictures of their empty villages, and Hitler’s family tree as a keepsake [Heim 1942]. In the ensuing years, Nazi authorities had other things on their minds. Eventually, the district of Lower Austria was occupied by the Red Army, which could find good use for the vast training opportunities filled with bunkers and artillery ranges. By 1955, the Allied occupation troops left Austria, but the evacuated region was not returned to its former dwellers. They had been scattered all over the district and were far too weak to succeed in their demands for a return. The small new Austrian army managed to keep the oversized training grounds for itself. Those abandoned houses which were still standing, after the years of Nazi and Soviet occupation, including Edelbach, were now flattened in a remarkably short time. The Austrian army had inherited an amazing amount of ammunition, and made a point of spending it lavishly by shelling the empty settlements. Today only the church of Döllersheim survives: its spire serves as a convenient mark for ranging artillery sights.

But during Leray’s years of internment he was daily faced with the vacant houses of a seemingly intact, *menschenfrei* Edelbach behind the barbed-wire fence. The chimneys did not smoke and the doors never opened. The window-panes had been replaced by planks. A poem on the front page of the *Canard en KG*, with the title *Le village ignoré*, describes the mute bell-tower of the deserted hamlet, and the silence broken only by the wind [Poll 1989]. And while the Nazi picture book acknowledges that when Edelbach had to be cleared out, some left it with a bleeding heart, the captive French poet imagines how his heart, far from bleeding, “jumps with joy on the day, known only to destiny,” when he is released and the forsaken village vanishes behind the firs.

The day known only to destiny was April 17, 1945. The camp had to be evacuated because the Red Army was perilously close. The *Wehrmacht* was by now out of gas and lorries. The *Blitzkrieg* days were over. The prisoners had to march, carrying their belongings on their backs. Some of the guards used bicycles, and their officers sat on underfed horses. The trek aimed for Linz, some 128 kilometers away to the west. The group covered, on aver-

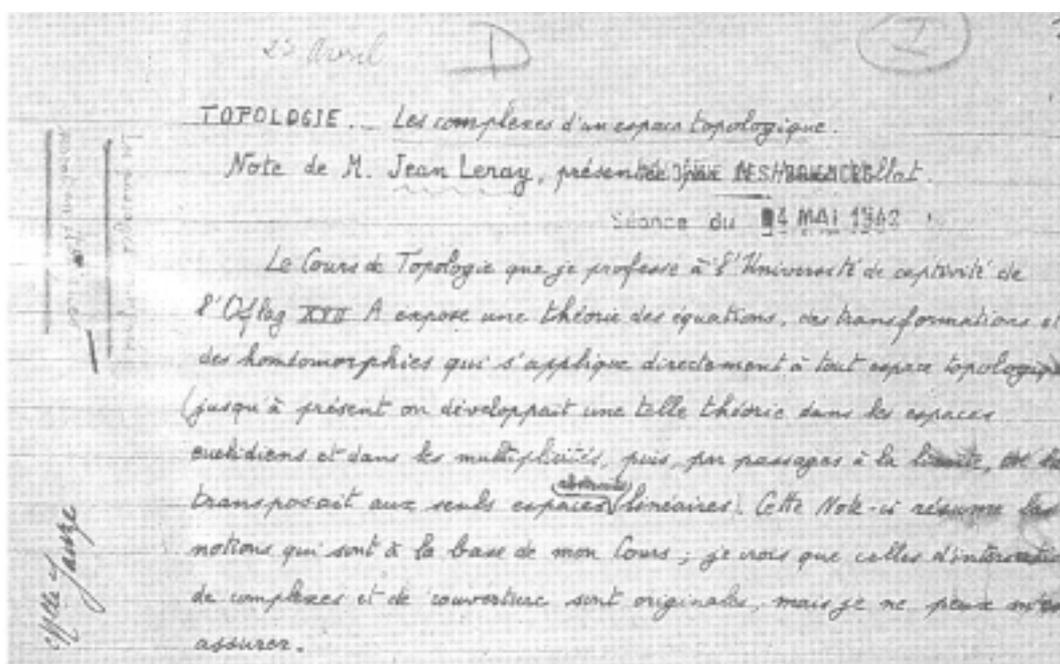


Fig. 4. Notes from captivity. KG Jean Leray reports, in this *Comptes Rendus* note from 1942, that in his present condition, he is unable to guarantee the originality of his results [Gaz 2000].



Fig. 5. No open university, but a closed universe of 440×530 meters. The camp, and campus, of Oflag XVII housed some 5,000 prisoners. Today, the barracks are gone: in their place one finds concrete, earth-covered ammunition dumps. The village of Edelbach is a rubble of stones covered by a dense forest.

age, less than ten kilometres a day, and dwindled rapidly in size. The marching column was long, the forest dense. Underfed François Ellenberger schlepped a rucksack half his own weight: he had insisted on taking along his voluminous mineralogical notes, a hand-made telescope, and his rock samples, some of which had come from the tunnel. He still found the strength to sketch the lines of the hills in his notebook, and

the interiors of rural chapels. The prisoners had to look after their own food; some managed to get it from old wives and barefoot children, in exchange for soap, which they had produced in their camp. By May 10, the column had been reduced by half. This was the day the *Wehrmacht* surrendered.

After his liberation, Jean Leray became professor, first at the University of Paris (which had appointed him in

1942), and then, in 1947, at the prestigious *Collège de France*. In 1953 he was elected to the *Académie des Sciences de Paris* (which had made him a corresponding member in 1944). He was showered with prizes: among them, the prix Ormoy in 1950, the Feltrinelli prize in 1971, the Lomonosov gold medal in 1988 (jointly with Sobolev), and in 1979, the Wolf prize, jointly with André Weil (who, incidentally, had also been



Fig. 6. A room with a view. The barracks were originally built for the *Wehrmacht* soldiers claiming the grounds. Fences and watchtowers were added later [Poll 1989].



Fig. 7. Underground film. These clandestine stills show the escape tunnel, also known as *métro pour la liberté*. The clandestine movie *Sous le drapeau*, shot in real time, is a French alternative to Hollywood's *The Great Escape* [Corr 1954].

a candidate for that same chair at the *Collège de France*). In an obituary written for *Nature*, Ivar Ekeland called Leray “the first modern analyst,” and compared him with Weil, “the first modern algebraist” [Eke 1999].

The parallels, which also were stressed by Jean-Michel Kantor [Gaz 2000], are indeed intriguing: the two men share their year of birth, 1906, and their year of death, 1998. They both were among the very select few to attend the *École Normale Supérieure*, and both did some of their best work in prison. But the differences are even more striking. Weil followed his

dharma (that is to say, he was a conscientious objector) and therefore took hair-raising risks to avoid waging war against Hitler. Leray served as a patriotic officer and remained stolidly at his post to the end, during the swift German assault and throughout the protracted years of confinement. Whereas Weil studied abstract algebraic structures and shunned anything even remotely smacking of applications or physical intuition, Leray was deeply steeped in physics and geometry. This makes all the more remarkable the fact that he switched to algebraic topology in the prison camp, and laid the basis

for what soon became a main item on Bourbaki's menu, although he had left the Bourbaki group in 1935.

Changing direction seems to have posed no problem for Leray. “The essential characteristic of my publications is their diversity,” he later said, simply. “It was my interest in mechanics that obliged me to give new developments to mathematical analysis and algebraic topology” [Sch 1990]. Indeed, Leray had been interested in topology even before the war, but as a tool rather than as an end in itself. The homotopy invariant now known as the Leray-Schauder degree was created in



Fig. 8. Cold feet and frosty advice. Unaware of being filmed, a Wehrmacht delegation decided to keep the news of the escape under wraps. But posters warned the French that henceforth, *s'évader n'est plus un sport*.



Fig. 9. The church of Edelbach, in an already deserted village. The poem laments that in the humble church, no bell ever rings. In 1957 the church was flattened by Austrian artillery.

order to prove the existence of solutions to non-linear partial-differential equations. Such equations, particularly those which stemmed from mathemat-

ical physics, were at the centre of Leray's work. In 1936, he published a truly pioneering paper investigating the existence, uniqueness, and smooth-

ness of solutions of the initial-value problem for the three-dimensional Navier-Stokes equations for incompressible fluids. He showed, in partic-



Fig. 10. Forty years after. This stone commemorates a visit in 1985 by some former inmates of the Oflag. The French prisoners had their own graveyard in Edelbach, complete with funeral statue.

ular, that non-stationary solutions for smooth initial data remain smooth for a finite time only; beyond this, they may only be continued in a weak sense (giving rise to what are called weak solutions nowadays). Leray called such solutions *turbulent*, thereby suggesting that the onset of turbulence is caused by the breakdown of smoothness. He certainly had good reasons not to wish the Germans to learn of his work. It is interesting to speculate what he would have done if he had been given an opportunity to do scientific work for the Allies.

As it was, he “turned his minor into his major interest” and started working on algebraic topology as an end in itself—Weil-style, as it were. He worked in great, but not total scientific isolation, avoiding contacts with German mathematicians. Apart from some reprints provided by Heinz Hopf, from neutral Switzerland, Leray was cut off from ongoing research, in particular from contemporary, related work by Eilenberg and Steenrod, and had to start from scratch.

As Armand Borel later wrote, Leray’s original concepts, based on a language of his own making, have been strongly modified or have not survived [BHL 2000]. Leray’s aim was to create something similar to differential forms, keeping their multiplicative algebraic structure, but in a purely topological framework. His cohomology was similar to that created by Čech, and his results did not, as Borel wrote, “seem to go drastically beyond those of mainstream algebraic topology.” But the intention behind them was different: Leray aimed at studying, not only the topology of a space, but the topology of a representation, i.e., topological invariants for continuous maps. He took as starting point his notes on a course by Élie Cartan on differential forms, published in 1935 [Cart 1935]. He aimed to understand cohomology (which he persistently called homology) in a way similar to the de Rham cohomology, with its multiplicative structure.

From his work with Schauder on fixed-point theorems, he was used to the relative viewpoint. He considered mappings between two spaces as

the basic object. This was a lasting achievement. The Leray-Serre spectral sequence of a filtration is still in general use today. Grothendieck would also stress the importance of the relative point of view in algebraic topology [Jack 2004].

Soon after his release, Leray found a way to define cohomology with respect to sheaves, and introduced the spectral sequence of a continuous map, which relates the cohomology of the domain to that of the range and of the fibre. His original ideas, intended to be as general as possible, were still not general enough, however, for three young Frenchmen named Henri Cartan, Jean-Louis Koszul, and Jean-Pierre Serre. They extended his concepts to obtain spectacular applications to analytic spaces and algebraic geometry. In the late forties, the development became almost breathless [Gaz 2000]. The two Fields medallists of 1954, Serre and Kodaira, both based their work on Leray’s sheaves and spectral sequences.

In the hands of Cartan and Oka, sheaves became an essential tool for the theory of several complex variables. Weil used sheaf cohomology and spectral sequences on real manifolds to give a lucid proof of de Rham’s theorem, generalising the Mayer-Vietoris sequence from an open cover of two sets to one of infinitely many sets. Godement wrote the definitive treatment of sheaves and their cohomology for algebraic topology. Serre and Grothendieck adapted the notion of sheaves for algebraic geometry. Even the (still unfinished) theory of motives concerns a category of sheaves. The central problem, on which Voevodsky made some recent inroads, is to find enough injective resolutions for cohomology to work. With the papers of Kodaira and Spencer, and the Habilitationsschrift of Hirzebruch [Hirz 1956], sheaf cohomology crossed the French borders. Sato used complex analytic sheaf cohomology to define hyperfunctions as generalised boundary values of holomorphic functions, and investigated microlocal analysis on the cotangent bundle. Sato’s microfunctions are more powerful than Hörmander’s wave-front sets, which in turn were inspired by Maslov. Later, Leray

would devote a whole book to the role of Planck’s constant in mathematics, again in an attempt to understand Maslov [Ler 1981].

Leray’s concept of spectral sequences appeared first as a complicated set of relations among various cohomologies of double complexes. They allowed Leray to compute the cohomology of compact Lie groups and flag manifolds. Serre used spectral sequences, already in their modern form, to determine the dimensions in which the higher homotopy groups of the n -sphere are not finite, namely n and $2n-1$. Massey made spectral sequences more easily accessible via the notion of exact couples.

Leray himself, after 1950, returned to partial-differential equations. He studied the Cauchy problem, its connection with multidimensional complex analysis, residue theory on complex manifolds, and integral representations. Algebraic topology became a tool again for Jean Leray. The interlude which had begun in the POW camp of Edelbach, as a kind of camouflage, was over. But generations of pure mathematicians would exploit the ideas which had germinated in Oflag XVIIIa.

Acknowledgements

We thank Hofrat Dr. Andreas Kusternig for a wealth of information on Oflag XVIIIa. Jean-Michel Kantor, Reinhard Siegmund-Schultze, and Hannelore Brandt provided considerable help in preparing this article.

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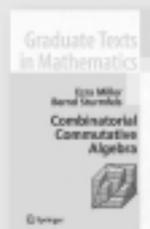
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