Scissors-and-paste with Λ: The geometric picture

Roland Steinbauer

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Overview

- General topic: exact sols. with a twist towards low regular metrics
- based on a series of joint papers with
  with Jiří Podolský, Clemens Sämann & Robert Švarc
- part of a broader line of research on

  Impulsive gravitational waves

  which are models of short but violent burst of gravitational radiation
- Why impulsive waves?

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  - particle physics: quantum scattering, wave memory effect

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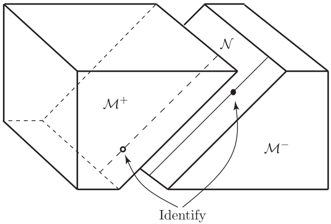
  - particle physics: quantum scattering, wave memory effect
- History: [Penrose, late 60s, early 70s] sicssors and paste approach
  [Aichelburg&Sexl, 72] ultrarel. boost of Schwarzschild
  [Hotta&Tanaka, 93] AS-boost with $\Lambda \neq 0$
  [Griffiths&Podolský, late 90s] systematic study for $\Lambda \neq 0$
  [PSŠS, 2014–] new geometric & mathematical insights

Cut & paste: explicit construction



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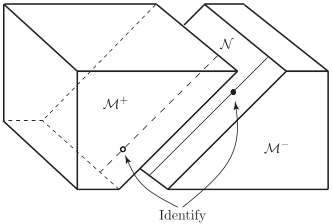

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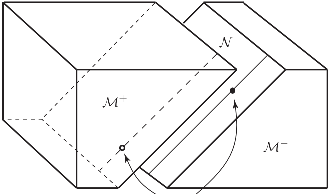

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$$\mathcal{U} = u$$
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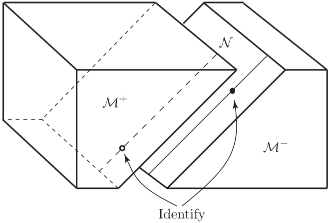

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- Apply (T) to (B) for $\mathcal{U} > 0, < 0$ separately $\rightsquigarrow$ **continuous form**

$$\mathrm{d}s^2 = 2\,|\mathrm{d}Z + u_+(h_{,Z\bar Z}\mathrm{d}Z + h_{,\bar Z\bar Z}\mathrm{d}\bar Z)|^2 - 2\,\mathrm{d}u\mathrm{d}v \qquad \text{(C)}$$

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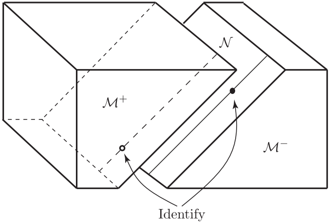

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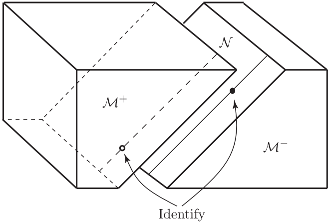

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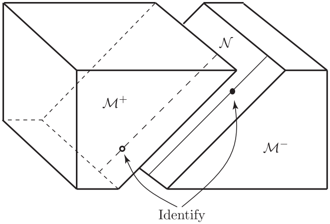

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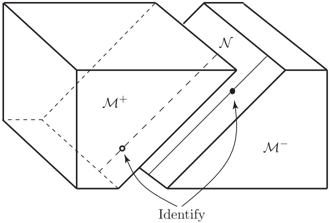

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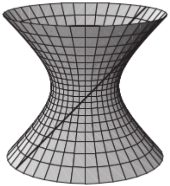
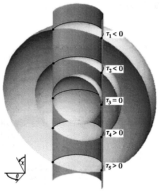


imp. wave in dS



propagating wave

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Questions

Q1) What happens to the cut & paste **picture**?

Q2) What is the meaning of the **'discontinuous transformation'** (T)?

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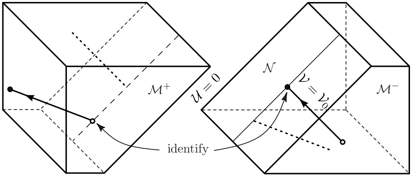
The ‘discontinuous transformation’ for $\Lambda = 0$



Cut & paste in $\mathcal{M}$ revisited

- **Key observation:**
  (T) is closely related to the null geodesics in (D)
- $\gamma(\mathcal{U}) = (\mathcal{V}, \eta)(\mathcal{U})$ with data $\gamma(-\infty) = (v, Z)$, $\dot{\gamma}(-\infty) = 0$
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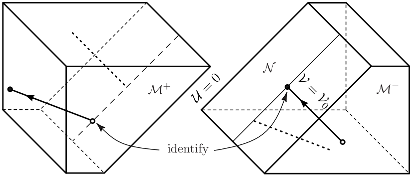

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Actual treatment of (D):
regularisation!

- regularise (D): $\delta \rightsquigarrow \delta_\varepsilon$
- geodesics γ_ε of (D_ε) naturally give geometric regularisation (T_ε) of (T)
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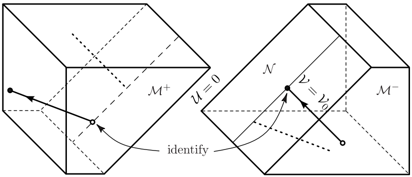

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Schematic picture

(D) lim $\longleftrightarrow$ reg. (D_ε)

(T) $\updownarrow$ $\qquad$ $\updownarrow$ (T_ε)

(C) lim $\longleftrightarrow$ reg. (C_ε)

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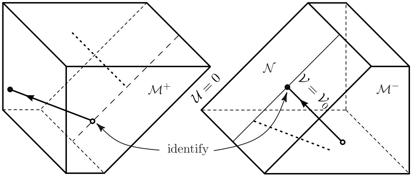

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Advanced math. treatment
needs fully nonlinear analysis of γ_ε

- global existence & uniqueness γ_ε cross wave impulse
- limits are broken backgr. geos
- (T) is limit of ‘generalised diffeo’ in nonlinear distr. geometry (Colombeau) [Kunzinger&S, 99]

The ‘discontinuous transformation’ for $\Lambda \neq 0$

- (null) geodesics in (D) are the key!
  i.e. interaction of the null particles with the wave impulse
- complicated nonlinear system with very(!) singular coefficients
- trick: use 5-**dim. representation** of (A)dS to tackle geodesic eq.
  (following [Podolský&Ortaggio, 01])
- Global existence and uniqueness result for regularised situation
  (using a fixed point argument)
  limits are again broken background geodesics [SSLP, 16]

The ... $\neq 0$

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  i.e...
- co... ts
- tr... eq.
  ... io, 01])
- G... n
  (u...
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Explicit jump formulas:

$$\gamma_{5D}(\lambda) = \begin{pmatrix} \lambda \\ V^0 + \dot{V}^0\lambda + \Theta(\lambda)\mathbf{B} + \mathbf{C}\lambda_+ \\ Z^0_p + \dot{Z}^0_p\lambda + \mathbf{A_p}\lambda_+ \end{pmatrix},$$

$$\mathbf{A_p} = \frac{1}{2}\Big(h^{\mathrm{i}}_{,p} + \frac{Z^0_p}{\sigma a^2}\left(h^{\mathrm{i}} - h^{\mathrm{i}}_{,q}Z^0_q\right)\Big), \qquad \mathbf{B} = \frac{1}{2}h^{\mathrm{i}},$$

$$\mathbf{C} = \frac{1}{8}\Big((h^{\mathrm{i}}_{,2})^2 + (h^{\mathrm{i}}_{,3})^2 + \sigma(h^{\mathrm{i}}_{,4})^2 + \frac{1}{\sigma a^2}\left({h^{\mathrm{i}}}^2 - (h^{\mathrm{i}}_{,p}Z^0_p)^2\right)\Big) + \frac{1}{2\sigma a^2}\left(h^{\mathrm{i}} - h^{\mathrm{i}}_{,p}Z^0_p\right)V^0 + \frac{1}{2}h^{\mathrm{i}}_{,p}\dot{Z}^0_p.$$

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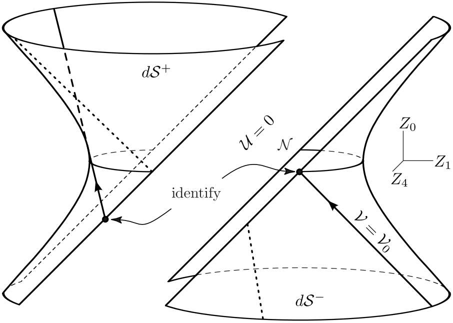
  limits are again broken background geodesics [SSLP, 16]
- nonlinear distributional analysis
  enables advanced mathematical treatment [SS, 17]

> **Answer to Q2)**
>
> (T) is the limit of a ‘generalised diffeomorphism’ in nonlinear distributional geometry (Colombeau). [SŠS, forthcoming]

- But where is the cut & paste picture?

Cut & paste with Λ: the geometric picture



Answer to Q1)

Interaction of null geodesic generators with the impulse reveals geometry of the cut & paste method: The generators

1. jump in $\mathcal{V}$ (due to Penrose junction conds.)
2. are refracted precisely to be null generators again

Some related literature

- R. Penrose. The geometry of impulsive gravitational waves. In General relativity. Clarendon Press, Oxford, 1972.
- M. Hotta M. Tanaka. Shock wave geometry with nonvanishing cosmological constant. CQG 10:307–314, 1993.
- J. Podolský, J. Griffiths. Nonexpanding impulsive gravitational waves with an arbitrary cosmological constant. PLA 261(1-2):1–4, 1999.
- M. Kunzinger, R. Steinbauer. A note on the Penrose junction conditions. CQG 16:1255–1264, 1999.
- J. Podolský, M. Ortaggio. Symmetries and geodesics in (anti-) de Sitter spacetimes with non-expanding impulsive waves. CQG 18(14):2689–2706, 2001.
- C. Sämann, R. Steinbauer, A. Lecke, J. Podolsky. Geodesics in nonexpanding impulsive gravitational waves with Λ, part I. CQG 33(11):115002, 2016.
- C. Sämann, R. Steinbauer. Geodesics in nonexpanding impulsive gravitational waves with Λ. II. JMP 58(11):112503, 2017.
- J. Podolský, C. Sämann, R. Steinbauer, R. Švarc. Cut and paste for impulsive gravitational waves with Λ: The geometric picture. PRD, to appear, 2019.
- C. Sämann, R. Steinbauer, R. Švarc. Cut and paste for impulsive gravitational waves with Λ: The mathematical analysis. in preparation, 2019.

El Fin — Muchas Gracias