The zero entropy locus for the Lozi maps

Joint work with M. Misiurewicz and S. Štimac

Kristijan Kilassa Kvaternik

University of Zagreb

36th Summer Topology Conference
Vienna, 2022

The Lozi map

- R. Lozi, *Un attracteur étrange du type attracteur de Hénon*, 1978
  - simpler model for the dynamics of the Hénon map
- 2-parameter family

$$L_{a,b} : \mathbb{R}^2 \to \mathbb{R}^2, \ \ L_{a,b}(x,y) = (1 + y - a|x|, bx)$$

- $a > 0, \ 0 < b \leqslant 1$
- 1980, M. Misiurewicz - strange attractor for the Lozi map

- $0 < b < 1$, $1 - b < a < 1 + b$
    - two hyperbolic fixed points

$$X = \left(\frac{1}{1+a-b}, \frac{b}{1+a-b}\right), \; Y = \left(\frac{1}{1-a-b}, \frac{b}{1-a-b}\right)$$

    - two attracting period-two points, P' in the 2nd and P in the 4th quadrant

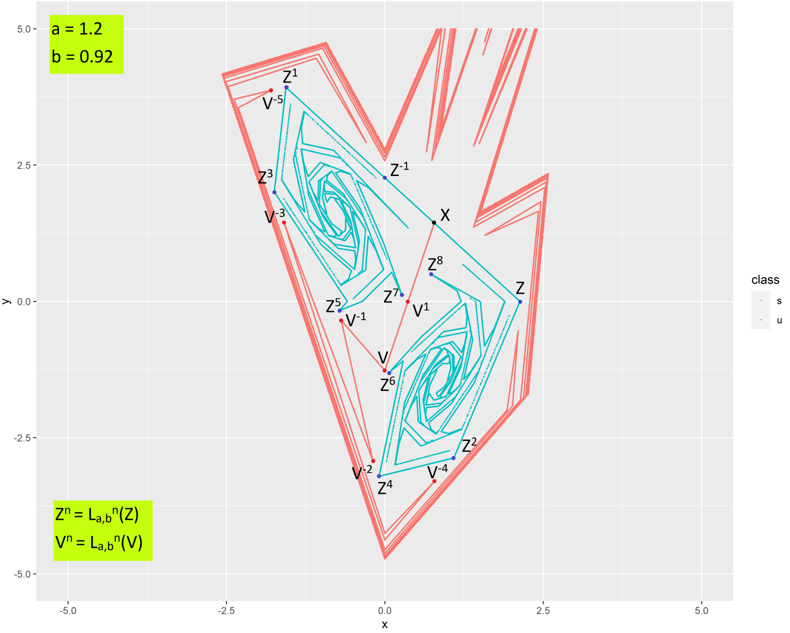
- invariant manifolds for X
    - **unstable manifold**

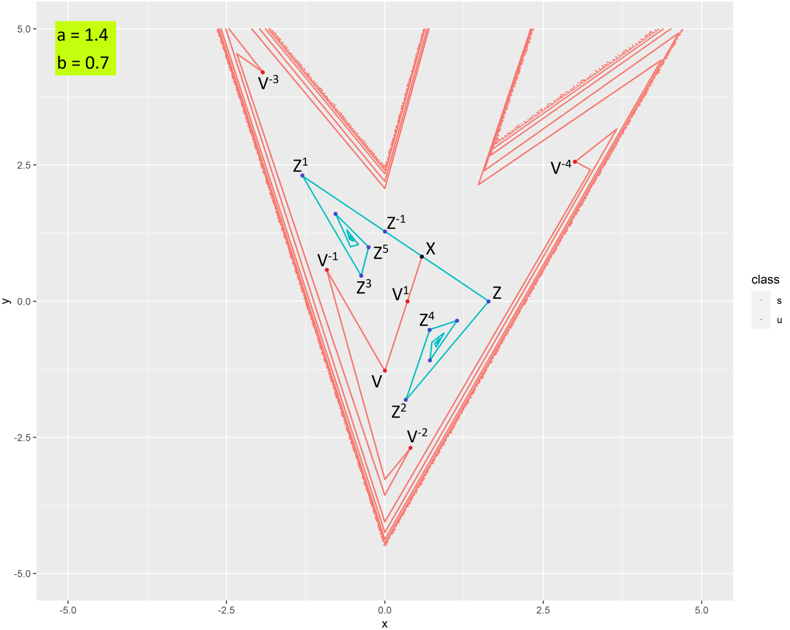
$$W_X^u = \{T \in \mathbb{R}^2 \colon L_{a,b}^{-n}(T) \xrightarrow{n\to\infty} X\}$$

    - **stable manifold**

$$W_X^s = \{T \in \mathbb{R}^2 \colon L_{a,b}^{n}(T) \xrightarrow{n\to\infty} X\}$$

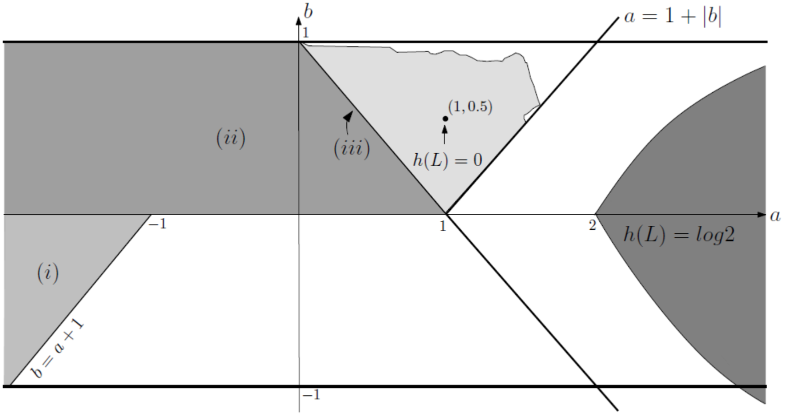
- $T \in W_X^u \cap W_X^s$ ... **homoclinic point** for X


a = 1.2
b = 0.92
$Z^n = L_{a,b}{}^n(Z)$
$V^n = L_{a,b}{}^n(V)$
Z^1
V^{-5}
Z^{-1}
Z^3
V^{-3}
X
Z^8
Z
Z^7
V^1
Z^5
V^{-1}
V
Z^6
Z^2
V^{-2}
V^{-4}
Z^4
5.0
2.5
0.0
-2.5
-5.0
y
-5.0
-2.5
0.0
2.5
5.0
x
class
s
u

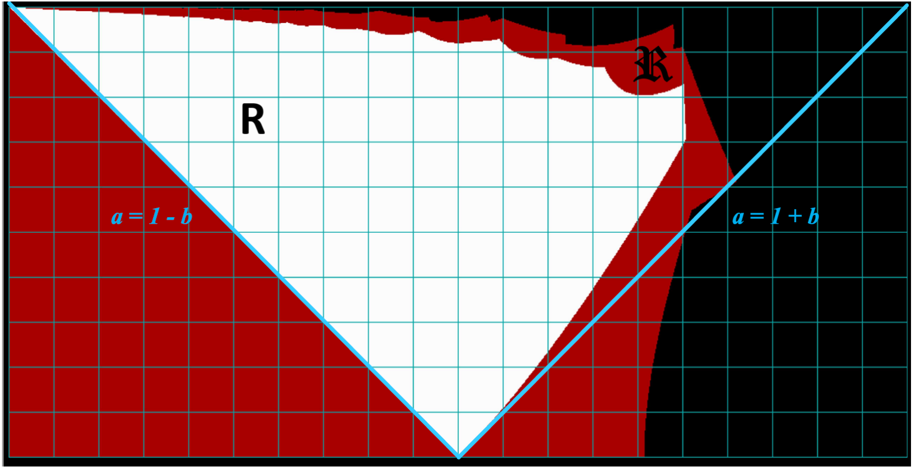

a = 1.4
b = 0.7
V-3
V-4
Z1
Z-1
X
Z5
V-1
Z3
V1
Z
Z4
V
Z2
V-2
class
s
u
y
x
5.0
2.5
0.0
-2.5
-5.0
-5.0
-2.5
0.0
2.5
5.0

What is known?

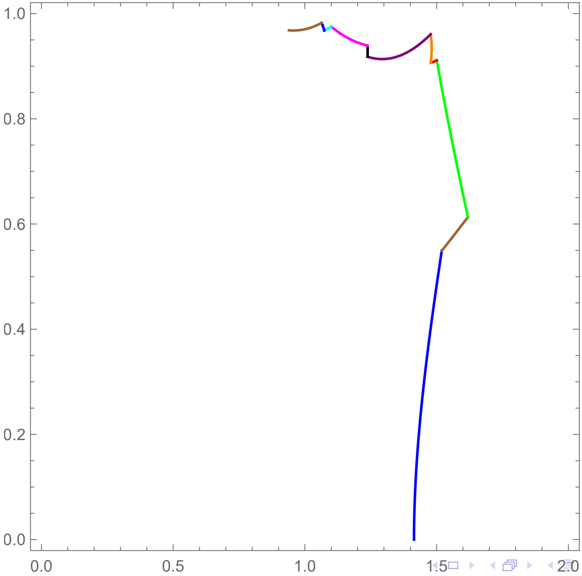
I. B. Yildiz, *Monotonicity of the Lozi family and the zero entropy locus*, 2011



Parameters of interest



Border of existence of homoclinic points for X



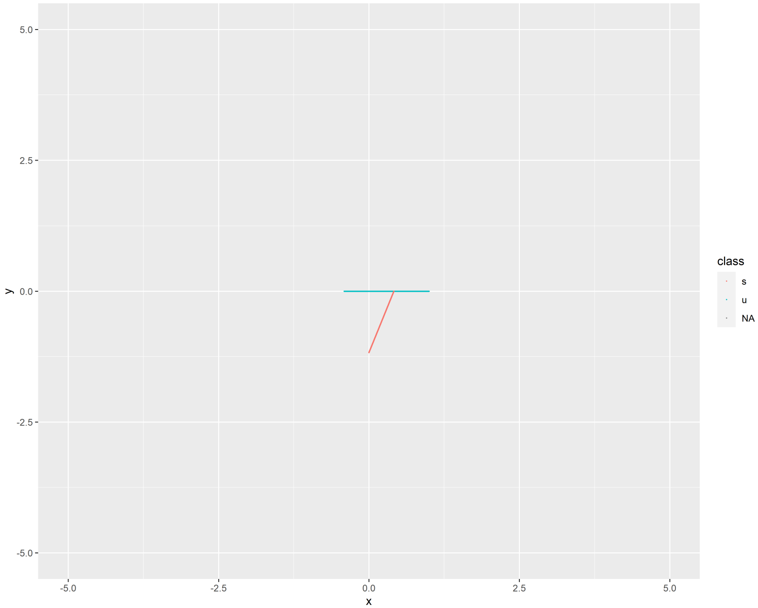
- Border curves

$$C_n \dots P_n(a,b) + Q_n(a,b)\sqrt{a^2+4b} = 0$$

$$\begin{aligned}
P_1(a,b) &= a^3 - 4a,\\
Q_1(a,b) &= a^2 - 2b,
\end{aligned}$$

$$\begin{aligned}
P_2(a,b) &= 4ab^5 - (8a^4-4)b^4 - (4a^5+8a^3+15a^2+4a+4)b^3 +\\
&\quad (15a^4+16a^3+11a^2)b^2 + (4a^7+2a^6-8a^5-10a^4)b\\
&\quad -(2a^8-2a^6),\\
Q_2(a,b) &= (-4a^4+4a^2-a)b^3 - (8a^4+a^3-3a)b^2 +\\
&\quad (4a^6+6a^5-6a^3)b - (2a^7-2a^5),
\end{aligned}$$

$$\begin{aligned}
P_3(a,b) &= 4b^3 + 3a^2b^2 - (a^4+6a^2+4a)b - (4a^4+4a^3+2a^2)),\\
Q_3(a,b) &= -3ab^2 - (a^3-2a)b + (4a^3+4a^2+2a), \dots
\end{aligned}$$

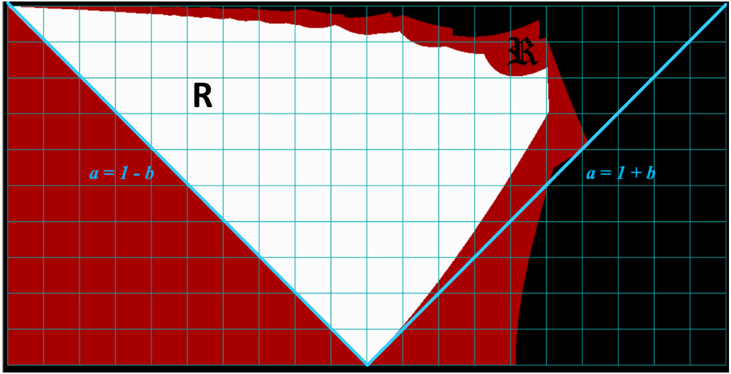


5.0
2.5
0.0
-2.5
-5.0
y
-5.0
-2.5
0.0
2.5
5.0
x
class
s
u
NA

Main result

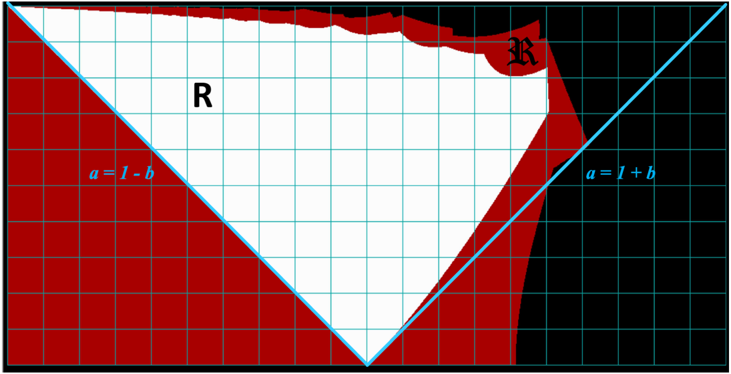
Theorem (KK, Misiurewicz, Štimac)

For all $(a, b) \in R$, $h_{top}(L_{a,b}) = 0$.



Work in progress

Is $h_{top}(L_{a,b}) = 0$ for all $(a, b) \in \mathfrak{R}$?



References

- Burns, K. and Weiss, H. (1995). A geometric criterion for positive topological entropy. *Communications in mathematical physics*, 172(1), 95-118.
- Ishii, Y. (1997). Towards a kneading theory for Lozi mappings I: A solution of the pruning front conjecture and the first tangency problem. *Nonlinearity*, 10(3), 731-747.
- Lozi, R. (1978). Un attracteur étrange (?) du type attracteur de Hénon. *Le Journal de Physique Colloques*, 39(C5), 9-10.
- Misiurewicz, M. (1980). Strange attractors for the Lozi mappings. *Annals of the New York Academy of Sciences*, 357(1), 348-358.
- Yildiz, I. B. (2011). Monotonicity of the Lozi family and the zero entropy locus. *Nonlinearity*, 24(5), 1613-1628.

THANK YOU!