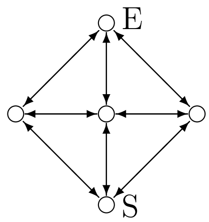
Exercise Sheet 1, Proseminar Stochastic Processes
Winter Semester 2016-17, 250069 PS

Exercise 1 *Let $A = (a_{ij})$ be an $n \times n$ probability matrix, i.e., $a_{ij} \geq 0$ and the row-sums $\sum_j a_{ij} = 1$. Show that*

1. *A^k is a probability matrix for every $k \in \mathbb{N} \cup \{0\}$;*
2. *1 is an eigenvalue of A;*
3. *there are no eigenvalues λ with $|\lambda| > 1$;*
4. *the left and right eigenvectors with eigenvalue 1 can be chosen such that $v_j \geq 0$ for all $j = 1, \ldots, n$.*

Exercise 2 *Consider the following transition graph, where from each node, each passage to a neighbouring node is equally likely.*



1. *Give the transition probability matrix associated to this graph.*
2. *Starting from node S, what is the probabilty of returning to S after three, resp. four steps?*
3. *A marmot starts in S and walks the graph until he reaches state E. What is the probability that he returns to S before reaching E?*
4. *What is the expected number of steps for the marmot to reach the end?*

Exercise 3 *Lisa and Bart play a game with one die and six marbles. They start with three marbles each, and roll the die. If the number of spots is 1 or 2, then Bart loses two marbles to Lisa. If the number of spots is 3,4,5 or 6, then Lisa loses one marble to Bart. They continue until one of them has all the marbles, who then of course is the winner.*

1. *What is the probability that Lisa wins?*
2. *What is the probability that Lisa wins after first getting down to one marble?*

Exercise 4 *Consider the difference equation*

$$y_{n+2} + ay_{n+1} + by_n = c, \qquad n \in \mathbb{N},\ y_n \in \mathbb{R},\ a, b, c \in \mathbb{R}.$$

1. *Find the full solution if $a = -2$, $b = -3$, $c = 0$ and initial condition $y_0 = y_1 = 1$.*
2. *Find the full solution if $a = -2$, $b = -3$, $c = 2$ and initial condition $y_0 = y_1 = 1$.*
3. *Find the full solution if $a = -2$, $b = 1$, $c = 0$ and initial condition $y_0 = 1, y_1 = 2$.*