Exercise Sheet 3, Proseminar Stochastic Processes
Winter Semester 2016-17, 250069 PS

Exercise 1 *Let* $P = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{pmatrix}$ *be the transition matrix of a Markov chain on state space* $E = \{1, 2\}$. *Compute the expected return time* $\mu_i = \mathbb{E}(T_i)$ *for* $i = 1, 2$. *What is the distribution of* T_i*?*

Exercise 2 *Given a finite irreducible Markov chain, show that all states are not just recurrent, but* **positive** *recurrent, i.e.,* $\mu_i = \mathbb{E}(T_i) < \infty$ *for the return time* T_i *to state* i*, and any* $i \in E$. *Show that* T_i *has finite variance as well.*

Exercise 3 *Let* P *be the transition matrix of a finite state Markov chain, and assume that* P *is irreducible, but periodic with period* $d \geq 2$. *Show that* E *decomposes into* d *communication classes for them* d*-th iterate of the process, i.e., for the process* $(X_{dn})_{n \geq 0}$. *Show that* $e^{2\pi ic/d}$ *is an eigenvalue of* P *for all integers* $0 \leq c < d$.

Exercise 4 *We are given an irreducible aperiodic Markov chain with finite state space* E*, transition matrix* P *and stationary distribution* π.

1. *Show that the first return process to a subset* E' *is also a Markov chain; what is the stationary distribution of this Markov chain?*

2. *If there are states* $i, i' \in E$ *such that* $p_{ij} = p_{i'j}$ *and* $p_{ji} = p_{ji'}$ *for all* $j \in E$*, show that we find a new Markov chain by merging states* i *and* i'*; what is the stationary distribution of this Markov chain?*