Exercise Sheet 4, Proseminar Stochastic Processes
Winter Semester 2016-17, 250069 PS

Exercise 1 *Let $(X_n)_{n\geq 0}$ be a random walk on the non-negative integers, with $p_{0,1} = 1$ and $p_{n,n-1} = p$, $p_{n,n+1} = 1 - p$ for $n \geq 1$. For which p is this random walk positive recurrent/null-recurrent/transient? In case of positive recurrence, what is the stationary distribution?*

Exercise 2 *Let G be an infinite rooted binary graph, i.e., there is a single root R and every other vertex has exactly three neighbours (so two "below" it), and there are no loops. Let $(X_n)_{n\geq 0}$ be a random walk on G.*

1. *Suppose, in one step you can only go to a neighbouring vertex, and from every vertex there is equal probability to jump to any of its neighbours. Is this random walk positive recurrent/null-recurrent/transient?*

2. *Same question, but now the probability to jump to the neighbour closer to the root is $\frac{1}{2}$ (and from the root you always jump to its single neighbour).*

Exercise 3 *Let $(X_n)_{n\geq 0}$ be a random walk on $\mathbb{Z}$, with transition probabilities $p_{n,n-2} = \frac{1}{4}$, $p_{n,n-1} = \frac{1}{4}$ and $p_{n,n+1} = \frac{1}{2}$. Is this random walk positive recurrent/null-recurrent/transient?*

Exercise 4 *Let $(X_n)_{n\geq 0}$ be a symmetric random walk on $\mathbb{Z}$ with $X_0 = 0$. Let $u_{2n} = \mathbb{P}(X_{2n} = 0)$ and $f_{2n} = \mathbb{P}(\min\{k \geq 1 : X_{2k} = 0\} = n)$.*

1. *Show that $f_{2n} = \frac{1}{2n-1} u_{2n}$.*

2. *Show that $u_{2n} = \sum_{k=1}^{n} f_{2k} u_{2(n-k)}$.*

3. *Show that $\sum_{k=0}^{n} u_{2k} u_{2(n-k)} = 1$.*