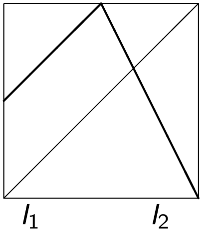
Henk Bruin (Surrey/Vienna)

# That monotonous thing called entropy

Mankato, July 2012

File of Figures

# Computing entropy via transtition matrices

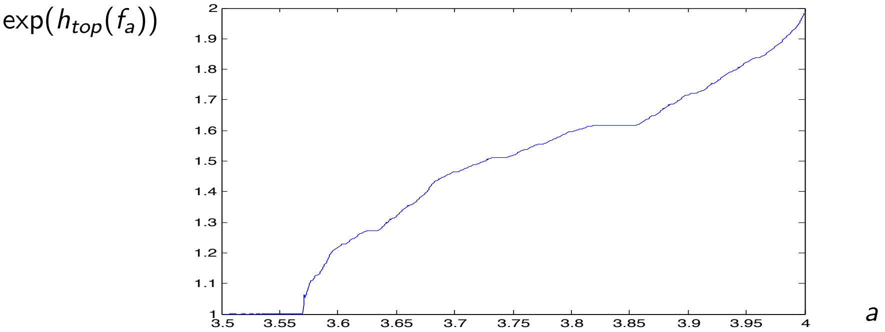



$$I_2 \circlearrowleft$$
$$\uparrow\downarrow$$
$$I_1$$

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\sigma(A) = \log \frac{1+\sqrt{5}}{2}$$

# $h_{top}(f_a)$ for quadratic family $f_a(x) = ax(1-x)$



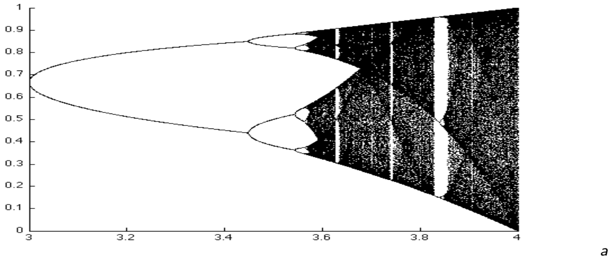



Figure: Bifurcation diagram for $f_a(x) = ax(1-x)$

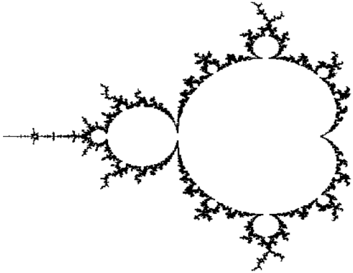


Figure: The Mandelbrot set $\mathcal{M}$

# Riemann maps and external rays

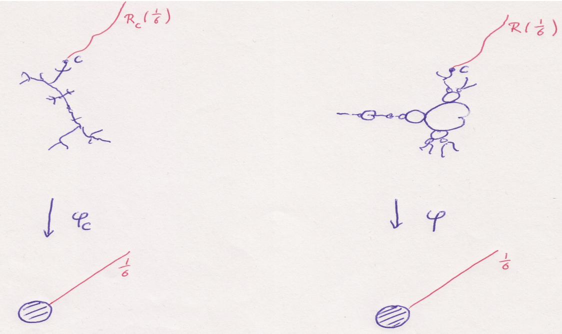



Figure: Riemann map $\phi$ and $\phi_c$ and external rays for angle $\frac{1}{6}$ for the filled-in Julia set and the Mandelbrot set.

# Riemann maps and external rays

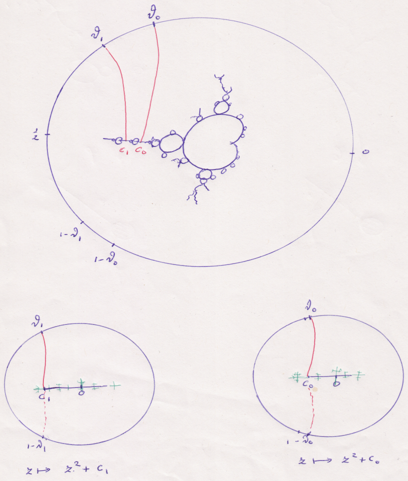



Figure: External rays $\theta_0$ and $\theta_1$ and corresponding rays for the Mandelbrot set and filled-in Julia sets.

# Hubbard tree for $\nu = 11010101\ldots$

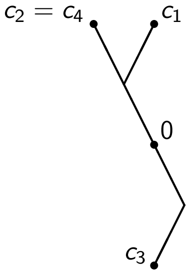
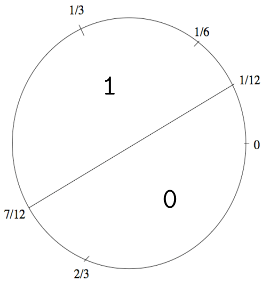
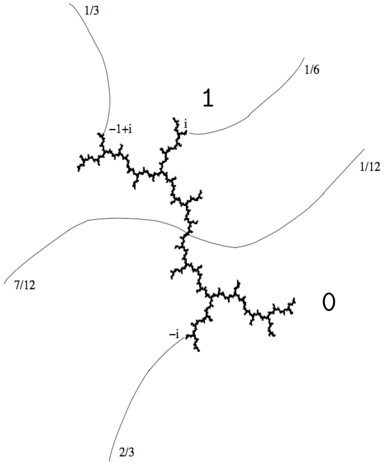


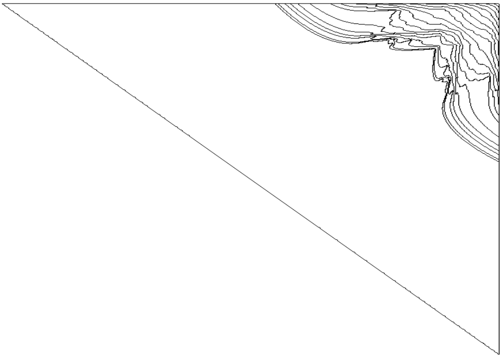

Figure: Hubbard tree for $\nu = 11010101\ldots$

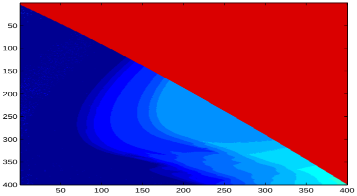
## Symbolic Dynamics for the Angle Doubling and Julia Sets



# Isentropes for the cubic family



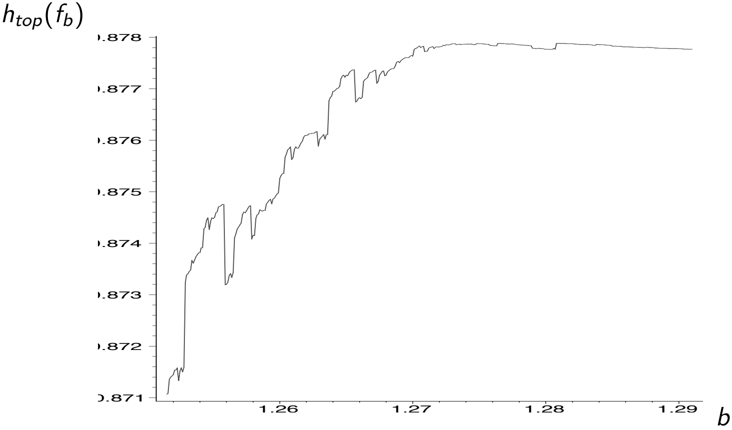



The cubic family

$$x \mapsto x^3 - ax + b.$$

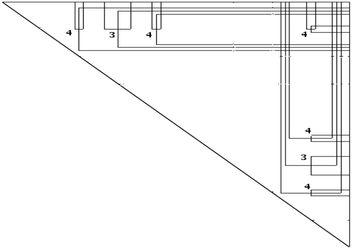
Isentropes in blue colour:

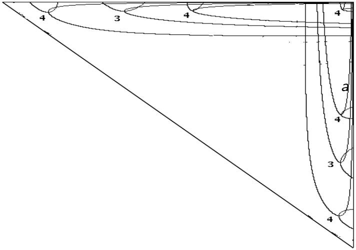
# Non-monotonicity of entropy in single ciritical value for cubics.



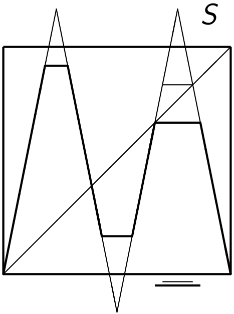
# Bones

Milnor and Tresser
analyse bifurcation curves,
see figures on the right.
They use planar topology
to show ‘bones’ are connected.



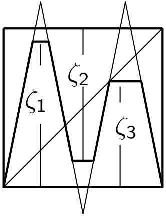



# Stunted Saw-Tooth Maps

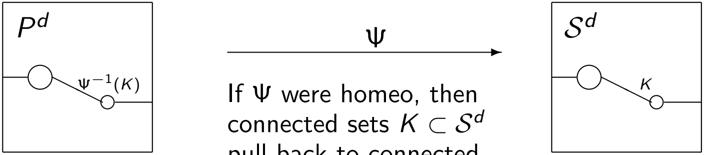



The saw-tooth map $S$

Two stunted sawtooth maps, with different third plateaus.



# The Map $\Psi : P^d \to \mathcal{S}^d$




If $\Psi$ were homeo, then connected sets $K \subset \mathcal{S}^d$ pull back to connected sets $\Psi^{-1}(K) \subset P^d$