IK-Seminar - 19,10,2006

"T-groups: Cohomology and polynomial crystallographic actions."

Key-words: *Homological algebra, cohomology of groups with coefficients, finitely generated, torsion-free, nilpotent groups: the so-called $\mathcal{T}$-groups; the Seifert-construction method; aspherical spaces, affine and polynomial crystallographic actions; Lie groups and Lie algebras; de Rham cohomology of invariant differential forms.*

First, I define the cohomology groups with coefficients in a module, using the language of homological algebra. As an important application, the second cohomology group is interpreted in terms of group extensions. The properties of the connecting homomorphism between the first and the second cohomology group are shown to be essential for the solution to the lifting problem for group actions. In fact, a vanishing theorem by Conner and Raymond, and another by Dekimpe and Igodt imply that every $\mathcal{T}$-group admits a relatively unique, smooth resp. polynomial crystallographic action of canonical type. Standard arguments of algebraic topology imply that every $\mathcal{T}$-group can be seen as the fundamental group of a compact, aspherical manifold.

There is a second way to obtain these polynomial crystallographic actions of $\mathcal{T}$-groups: the famous theorems of Malcev for $\mathcal{T}$-groups and their Malcev completions. One of the advantages of this construction method is that it is easy to work with for practical purposes, unlike the first method. Several examples are presented to illustrate both methods.

Finally, the crystallographic actions are used to compute the cohomology of $\mathcal{T}$-groups with trivial, real coeffcients. This cohomology is isomorphic to the cohomology of differential forms that are invariant under the induced action of the Malcev completion of the $\mathcal{T}$-group. It is then relatively simple to compute these invariant differential forms explicitly, and thus the cohomology of the original group. Again, this calculation is applied to a number of examples.