Minimal faithful representations of reductive Lie algebras.

Wolfgang Moens

10.05.2007

We consider the minimal dimension $\mu(\mathfrak{g})$ of a faithful $\mathfrak{g}$-module for finite-dimensional complex Lie algebras. By Ado's theorem, $\mu(\mathfrak{g})$ is an integer-valued invariant of $\mathfrak{g}$. For certain classes of Lie algebras, $\mu(\mathfrak{g})$ can be determined explicitly. For example, for an abelian Lie algebra $\mathfrak{g}$, the invariant $\mu(\mathfrak{g})$ can be determined by using classical results of Schur and Jacobson. The case of simple Lie algebras can be solved by highest weight theory, and the case of reductive Lie algebras follows from calculating centralizers of representations, together with combinatorical arguments. As a corollary we obtain a method to classify all reductive subalgebras of $\mathfrak{gl}_n(\mathbb{C})$. Dynkin, Borel and Siebenthal classified all proper maximal reductive subalgebras of all semisimple Lie algebras. This gives us a second (but in general very complicated) way to calculate $\mu(\mathfrak{g})$ for reductive Lie algebras. On the other hand, the invariant $\mu(\mathfrak{g})$ gives us some information about Dynkin's list. This is also related to Malcev's computations for the maximal dimension of abelian subalgebras of semisimple Lie algebras.