Smooth multiparameter perturbation of polynomials and operators

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Abstract: Let $P(x)(z) = z^n + \sum_{j=1}^n (-1)^j a_j(x) z^{n-j}$ be a family of polynomials whose coefficients a_j are smooth complex valued functions defined near $0 \in \mathbb{R}^q$. For generic P we construct a finite collection $\mathcal{T}$ of transformations $\Psi : \mathbb{R}, 0 \to \mathbb{R}, 0$, such that

$$\bigcup \{\mathrm{im}(\Psi) : \Psi \in \mathcal{T}\}$$

is a neighborhood of 0, which desingularizes the roots of P, i.e., for each $\Psi \in \mathcal{T}$, the roots of $P \circ \Psi$ allow smooth parameterizations near 0. As a consequence we prove that generic P locally admit roots with first partial derivatives in L^1. Moreover, we deduce a simple proof of Bronshtein's theorem (under slightly stronger conditions): For hyperbolic (not necessarily generic) P with $C^{n(n+1)/2}$ coefficients any continuous arrangement of its roots is locally Lipschitz. There are applications to the perturbation theory of linear operators.