The convenient setting for Denjoy-Carleman ultradifferentiable functions and its applications in infinite dimensional differential geometry

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Abstract. Based on:
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Let $M = (M_k)_{k\in\mathbb{N}}$ be an increasing sequence ($M_{k+1} \geq M_k$) of positive real numbers with $M_0 = 1$. Let $U \subseteq \mathbb{R}^n$ be open. We denote by $C^M(U)$ the set of all $f \in C^\infty(U)$ such that, for all compact $K \subseteq U$, there exist positive constants C and ρ such that

$$\boxed{\frac{|\partial^\alpha f(x)|}{\rho^{|\alpha|}\,|\alpha|!\,M_{|\alpha|}} \leq C}$$

for all $\alpha \in \mathbb{N}^n$ and $x \in K$. The set $C^M(U)$ is the *Denjoy–Carleman class* of functions on U. If $M_k = 1$, for all k, then $C^M(U)$ coincides with the ring $C^\omega(U)$ of real analytic functions on U. In general, $C^\omega(U) \subseteq C^M(U) \subseteq C^\infty(U)$. Recently we suceeded to establish convenient calculus for some of these function classes. There are many immediate applications in infinite differential geometry, and I will present two of them.