In this thesis we study the so-called μ-invariant of Lie algebras. For a finite-dimensional Lie algebra $\mathfrak{g}$, it is the minimal dimension of a faithful $\mathfrak{g}$-module. Showing that this invariant is finite, i.e. showing that every finite-dimensional Lie algebra has a finite-dimensional faithful module, is already a non-trivial problem. This fundamental result was originally proved by Ado and Iwasawa, and it has a long history. This thesis is about refining Ado's theorem in the following way:

Let $\mathfrak{g}$ be a finite-dimensional Lie algebra. Compute $\mu(\mathfrak{g})$ and find a faithful module of this dimension. Obtain upper and lower bounds for $\mu(\mathfrak{g})$ in function of other invariants. Describe the properties of faithful modules of minimal dimension.

We cannot expect an explicit formula in general, not even for the class of nilpotent Lie algebras. One can then ask whether it is possible to determine the invariant for reductive Lie algebras.

We obtain a formula for $\mu(\mathfrak{g})$ if $\mathfrak{g}$ is abelian, simple, semisimple and reductive. The proof is combinatorical in nature and uses classical results about representations for reductive Lie algebras. More generally, we study the μ-invariant of Lie algebras with a solvable radical that is in fact abelian. We consider other invariants that are related to the μ-invariant.