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Requirement of optical coherence for continuous–variable quantum teleportation

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We show that the sender (Alice) and the receiver (Bob) each require coherent devices in order to achieve unconditional continuous variable quantum teleportation (CVQT), and this requirement cannot be achieved with conventional laser sources, linear optics, ideal photon detectors, and perfect Fock state sources. The appearance of successful CVQT in recent experiments is due to interpreting the measurement record fallaciously in terms of one preferred ensemble (or decomposition) of the correct density matrix describing the state. Our analysis is unrelated to technical problems such as laser phase drift or finite squeezing bandwidth.

Quantum teleportation, first proposed as a method for teleporting an unknown spin–1/2 quantum state from a sender, "Alice", to a receiver, "Bob" [1], has been extended to continuous–variable quantum teleportation (CVQT) [2–5]. In this variant of quantum teleportation, Alice and Bob share a nonlocal entangled two–mode field which, ideally, should have perfect correlations between in–phase quadrature ('position' x) and out–of–phase quadrature ('momentum' p), although the requirement of perfect correlations can be relaxed somewhat [3]. Unconditional teleportation of coherent states was recently claimed to have been experimentally demonstrated [4], with two–mode squeezed light providing the nonlocal entangled resource shared between Alice and Bob. Although experimental advances towards CVQT have been achieved, we show that genuine CVQT cannot be achieved using conventional laser sources, due to an absence of intrinsic optical coherence [6]. This incoherence is not due to phase drift but is rather a consequence of entanglement between the laser medium and the output field. Herein we exclusively use the term coherence to refer to coherent superpositions of Fock states. In this letter, we establish that optical coherence is *required* for true CVQT to succeed, that conventional lasers do not meet this criterion, and therefore that unconditional CVQT is not possible with conventional lasers.

Clear criteria for evaluating the success of a teleportation experiment have been established [5], and we emphasize three relevant criteria for determining whether CVQT is possible at all with conventional laser sources: (a) the states to be teleported should be unknown to Alice and Bob and supplied by an actual third party Victor; (b) Alice and Bob share *only* a nonlocal entangled resource and a classical channel through which Alice transmits her measurement results to Bob; and (c) entanglement should be a verifiable resource. In particular we will show that the experiment of Ref. [4] does not satisfy any of these criteria and establish that any scheme based on conventional laser sources cannot simultaneously satisfy all three criteria.

The ideal scheme for CVQT [3] is explained in Fig. 1(a), and a schematic and explanation of the experimental simulation of CVQT is provided in Fig. 1(b). We emphasize that the scheme of Fig. 1(a) presupposes the availability of large amplitude coherent states to serve as local oscillators (LOs) for both Alice and Bob, and as a pump to produce the two-mode squeezed state. One should also assume that Victor has the ability to produce truly coherent states. In the experimental simulation of CVQT [4], the same laser source is used for (i) producing Victor's state for teleportation, (ii) pumping the nonlinear crystal that produces a two–mode squeezed light field serving as the shared Einstein-Podolsky-Rosen (EPR) state, (iii) supplying the LO fields for both of Alice's homodyne measurements, and (iv) providing Bob with a coherent field to mix with his portion of the EPR beam to reconstruct Victor's state, as depicted in Fig. 1(b).

In all discussions relating to optical CVQT, lasers have been presumed sources of (Gaussian) coherent states [7] $|\alpha e^{i\phi}\rangle = \exp(-\alpha^2/2)\sum_{n=0}^{\infty}(\alpha^n e^{in\phi}/\sqrt{n!})|n\rangle$. In quantum optics it is, in fact, standard to assume that the output of an ideal single–mode laser operating well above threshold is given by a coherent state. However, it is also well known that the phase ϕ of this laser field is completely unknown (not to be confused with the physically irrelevant global phase of a wavefunction). As such, the state of the laser output field ρ_L *must* be represented by a density matrix with phase eliminated by integrating over the distribution representing this lack of classical information. The resulting density matrix,

$$\rho_L = \int_0^{2\pi} \frac{d\phi}{2\pi}\,|\alpha e^{i\phi}\rangle\langle\alpha e^{i\phi}| \tag{1a}$$

$$= e^{-\alpha^2}\sum_{n=0}^{\infty}\frac{\alpha^{2n}}{n!}|n\rangle\langle n|\ , \tag{1b}$$

is the correct quantum mechanical description of an ideal laser mode output. As ρ_L is diagonal in the energy eigenstate (Fock) basis it is, in our precise terminology, not coherent.

It is common in quantum optics to assume that the single–mode laser field is in a coherent state $|\alpha e^{i\phi}\rangle$ to a good approximation. From Eq. (1a) it is clear why this works so well: this particular decomposition (or ensemble) of the reduced density matrix for the laser light corresponds to a mixture of coherent states with equally likely phase, and the success or failure of experiments in quantum optics is never dependent on knowing the *presumed* initial phase of the laser. More precisely, conventional measurements on optical fields, such as homodyne detection using lasers, involve mixing of different incoherent fields and subsequent detection by energy absorption in photodetectors: all such measurements are completely insensitive to any optical coherences [8].

We have mentioned the entanglement between the laser field and the state of the lasing medium as the source of the laser field phase indeterminacy. In [6] Mølmer explains carefully the absence of optical coherence for conventional lasers and produces a convincing analysis that shows optical coherence is a "convenient fiction" in quantum optics experiments. His analysis proceeds as follows. The gain medium of a laser consists of distinguishable dipoles (e.g. atoms), which are initially in a thermal distribution of their energy eigenstates and, therefore, initially possess a mean dipole moment of zero. This gain medium is incoherently pumped (the pumping field too is in a classical distribution of energy eigenstates and therefore has a zero mean electric field), and the standard interaction between the atoms and this field produces a large *entangled* state of the laser mode, pumping modes, gain medium and so on (with of course the build up of a large number of photons in the laser mode by stimulated emission). However, we are generally interested only in the light in the laser mode, and the correct description of this field state is via the reduced density matrix obtained by tracing out all other degrees of freedom. All processes involved are energy conserving, and the initial states of all participating physical systems (gain medium, pump modes, etc.) begin with no coherence: they are initially described by diagonal density matrices in their energy eigenstate bases. Evidently, tracing out the gain medium, pump modes, etc. of the large entangled state produced under these conditions *cannot* leave the laser mode in a state with any non-zero off-diagonal elements in its density matrix, let alone in a pure coherent state. More concisely, one cannot obtain states with quantum uncertainty in energy (coherent superpositions of Fock states) via physical processes which have only classical uncertainty in energy (that is, are diagonal in the Fock basis).

The intrinsic indeterminacy of the laser phase is unrelated to the technical issue of phase drift: phase stabilization methods via mixing of laser beams do *not* establish the existence of a phase for the laser. The importance of controlling phase drift is that *relative* phases of laser beams, generally (but not exclusively) spawned from the same source as in Fig. 1(b), have reduced drifts. This amounts to stabilization of the mode under consideration and is important because quantum optical interference experiments rely on *indistinguishability* (not coherence) for their success. Stabilizing this phase drift cannot introduce coherences into the overall density matrix.

The decomposition (1a) to coherent states notwithstanding, selecting one coherent state as the de facto state of the laser is clearly a commission of the "partition ensemble fallacy" (PEF) [9]. Within the framework of standard quantum mechanics, the density matrix is the complete description of the quantum state [10], and there is no reason to accord preferential treatment to one particular decomposition of the infinite number of equivalent decompositions for any mixed state. Avoiding preferential decompositions is clearly important for quantum information processing, which should be formulated operationally. We emphasize that the interpretation of experimental data under the assumption that the "real" state of the laser is actually one element of one decomposition of ρ_L yields a fallacious interpretation of the measurement record in terms of "what really happened".

Tomographic reconstruction of Gaussian states with well–defined phase [11] provides a simple example of how a failure to formulate operationally can lead to incorrect retrodiction of results. Such experiments generally rely on autocorrelation measurements in which the reconstructed state and the LO used for the homodyne measurements are derived from the same laser source, similar to the CVQT case considered in Fig 1(b). As the a priori assumption is that the laser produces a coherent state, the experimental data is interpreted in this context, and the tomographically reconstructed state is, therefore, interpreted as a coherent state. Since decomposition (1a) of ρ_L allows one to describe the initial laser state as a coherent state, and the experiment relies only on phase differences and not the overall initial phase of the laser, clearly no contradiction with this partitioning assumption can be achieved. It is convenient to assume that the laser field is in a coherent state, but such tomographic reconstruction does not "prove" this.

We are of course *not* asserting that production of coherent states of light is impossible: basic quantum electrodynamics shows that a classical oscillating current can produce coherent states [7]. As a classical current can be fully measured without disturbance, it provides the classical information necessary to specify a unique coherent state. Production of true coherent states is also possible if coherent measurements can be performed on one component of an entangled system. For example, a measurement on a two-level atom (with energy eigenstates $|g\rangle, |e\rangle$), coupled to a single photon cavity mode (with energy eigenstates $|0\rangle, |1\rangle$) in the entangled state $(|g,1\rangle + |e,0\rangle)/\sqrt{2}$, can be used to project the photon into a state with coherence, e.g. $(|0\rangle + |1\rangle)/\sqrt{2}$. However, such a measurement cannot be in the energy eigenstate basis of the atom, it necessarily requires coherence. Similarly, a coherent measurement on the gain medium of the laser, while technically challenging, could in principle project the laser output into a coherent state. However, processing the output of the conventional laser via linear optics, ideal photon detectors and perfect Fock state photon sources *cannot* be used to project the laser mode into a coherent state. CVQT as envisioned in all optical proposals thus far has been teleportation of phase and amplitude information of a coherent field. We therefore assert that genuine CVQT requires *coherent devices*, that is, devices capable of generating true coherence, and these are not a feature of current CVQT experiments.

We address the three criteria for successful CVQT, beginning with criterion (a). Splitting a coherent state $|\alpha e^{i\phi}\rangle$ at a lossless beam splitter with reflectivity r and transmissivity t yields a product state $|ir\alpha e^{i\phi}\rangle \otimes |t\alpha e^{i\phi}\rangle$, and the two output modes are, therefore, independent; one mode could be used by Victor. However, if the input to the beamsplitter is ρ_L, the resultant state is *not* of the form $\rho_1 \otimes \rho_2$ but rather

$$\begin{aligned}\rho_{\rm BS} &= \int_0^{2\pi} \frac{d\phi}{2\pi} |ir\alpha e^{i\phi}\rangle\langle ir\alpha e^{i\phi}| \otimes |t\alpha e^{i\phi}\rangle\langle t\alpha e^{i\phi}| \\ &= e^{-\alpha^2} \sum_{m,n=0}^{\infty} i^{n-m} \frac{(r\alpha)^{n+m}}{\sqrt{n!m!}} \sum_{r,s=0}^{\infty} \frac{(t\alpha)^{r+s}}{\sqrt{r!s!}} \\ &\quad \times \delta_{m+r,n+s}\, |m\rangle\langle n| \otimes |r\rangle\langle s| \ , \end{aligned} \tag{2}$$

with significant correlation between the two output modes. This clearly violates criterion (a). Decomposing state (2) to a specific product coherent state, with one of these two coherent states to be teleported, commits, once again, the PEF [9].

Criterion (b) stipulates that Alice and Bob should share *only* a nonlocal entangled resource and a classical communication channel. As the laser phase is indeterminate (more precisely it is a property whose "existence" depends on ascribing reality to one ensemble of ρ_L), Alice cannot include information on the phase of her laser source in the classical information she transmits to Bob, using linear optics, ideal photon detectors, and perfect Fock state sources. In the existing experiment Alice shares with Bob a laser beam split from the same source. He uses this beam to apply a unitary displacement operation (determined by Alice's transmitted classical information) to reproduce Victor's state. Without sharing this beam (or something similar but equally incoherent such as two slave beams locked to one master laser), Alice's and Bob's operations are completely uncorrelated. We maintain therefore that the absence of intrinsic optical coherence in a conventional laser necessitates the existence of this manifestly quantum channel. A classical channel cannot replace this because, although from the first decomposition of Eq. (2) ρ_{BS} is separable, it is locally preparable *only if* Alice and Bob each have independent coherent sources. To see this, note that if Alice and Bob each have only non–coherent devices (as defined earlier), classical communication only permits them to prepare mixed states (in the Fock basis), of the form $\rho = \sum_{ij} c_{ij} |i\rangle\langle i| \otimes |j\rangle\langle j| \neq \rho_{BS}$. In the experiment of Ref. [4], the majority of photons in the teleported state that Bob supplies to Victor (for verification) actually arrive from this extra quantum state shared between Alice and Bob. Without this shared field, the "convenient fiction" of coherence cannot be maintained. This quantum channel is in fact necessarily capable of transmitting the state Victor provided to Alice on its own; Victor would certainly be entitled to be suspicious of such a scheme. Teleportation as envisioned in [1] clearly requires that the only shared quantum state is the entangled state and Bob applying a unitary operation with only a classical device. In effect, without truly coherent devices the analogue of the single qubit rotations in [1] cannot be performed on arbitrary single mode states of light.

We now consider criterion (c). Conventionally in quantum optics, it is assumed that the nonlinear crystals used to produce squeezed states are pumped by a strong, pure coherent state. This results in a two–mode squeezed field,

$$|\eta e^{i\phi}\rangle = \sqrt{1-\eta^2} \sum_{n=0}^{\infty} \eta^n e^{in\phi} |n\, n\rangle \ , \tag{3}$$

with ϕ the phase of the pump field. However, the pump field actually originates from ρ_L of Eq. (1); hence, the two–mode output state from the nonlinear crystal is given by

$$\rho_S = \int_0^{2\pi} \frac{d\phi}{2\pi} |\eta e^{i\phi}\rangle\langle \eta e^{i\phi}| = (1-\eta^2) \sum_{n=0}^{\infty} \eta^{2n} |n\, n\rangle\langle n\, n| \tag{4}$$

which is clearly diagonal in the Fock state basis. It is therefore separable; Alice and Bob can prepare the state locally and consequently violate criterion (c). That ρ_S is diagonal arises because the pump field is diagonal in the Fock basis (i.e., $\rho_P = \sum \varrho_n |n\rangle_P\langle n|$), and thus has a classical energy spread or uncertainty. A nonlinear crystal pumped by a state with no energy uncertainty (i.e. $\rho_P = |N\rangle_P\langle N|$) transforms the pure state input $|N\rangle_P|0\,0\rangle$, with the two EPR fields in a vacuum state, into a superposition of states $\{|N-m\rangle_P|m\,m\rangle\}$. Tracing over the pump field state ρ_P destroys coherences in the reduced density matrix for the two squeezed modes, and results in the separable state (4). A two–mode squeezed state may *appear* to be pure, for example in tomographic reconstructions using balanced homodyne detection [12]. However, such reconstruction of a two–mode Gaussian (squeezed) state is another example of an invalid retrodiction by not properly applying the operational formulation, as discussed earlier. Thus, the pure state representation of the two–mode field as an entangled state is, in Mølmer's terminology, "a convenient fiction".

While we have focused here on homodyne type experiments such as [4], Molmer's argument applies equally well to systems of independent lasers interacting through conventional quantum optics devices. Such experiments cannot create nor demonstrate single mode coherence [6]. We assert, therefore, that genuine CVQT will require either new proposals not based on an assumption of coherence or new sources of optical radiation.

We conclude by describing how aspects of the experiment of [4] would be viewed if, instead of ascribing reality to the first decomposition (1a) of ρ_L, we equally fallaciously ascribe reality to the second decomposition (1b). We would then assert that the laser is in a pure number state $|N\rangle$ (although we are unsure exactly which one). When a pure number state encounters a beam splitter it produces an entangled state. Thus we would conclude that Victor's state is entangled with both Alice's and Bob's LOs. These in turn are entangled with the down-conversion pump beam (as described above), and this is entangled with the two squeezed modes. The experiment could now be claimed as a demonstration of an assortment of entanglement effects, although certainly not CVQT. Since the two decompositions of (1) describe exactly the same density matrix this description cannot be contradicted by the measurement record. Our view of course is that both this and the claimed interpretation are equally incorrect.

In summary, we have established that unconditional CVQT as claimed in Ref. [4] is not possible using conventional lasers because these are not genuine coherent sources. CVQT as envisioned in all optical proposals thus far has been teleportation of phase and amplitude information of a coherent field. The requirement of true optical coherence is our central result. We have shown that the experimental simulation of CVQT in [4] cannot be readily considered "unconditional teleportation" without committing the partition ensemble fallacy (PEF). Three criteria for claiming unconditional CVQT [5] have been shown to be violated for experiments using conventional laser sources.

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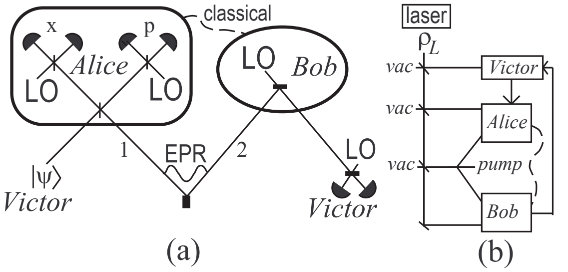

FIG. 1. (a) Ideal CVQT. Alice mixes, at a 50/50 beam splitter, Victor's state with her portion (component 1) of a two–mode squeezed vacuum state that is shared with Bob (component 2). In the ideal limit, the two–mode squeezed field is a pure entangled state with a Wigner function proportional to $\delta(x_1+x_2)\delta(p_1-p_2)$, for x_i (p_i) the in–phase (out–of–phase) quadrature of field component $i \in \{1,2\}$, thereby acting as a true EPR state. Alice mixes the two beamsplitter output fields with local oscillators (LOs) assumed to be in (infinitely strong) coherent states, thereby yielding true (x,p) quadrature–phase homodyne measurements. She transmits outcome (x,p) to Bob via a classical channel, which he uses to transform component 2 of the EPR state by mixing with a LO at a beam splitter. (b) An experimental simulation of CVQT using an incoherent (laser) source such as that adopted in Ref. [4]. The laser field is shared by all parties.