

Near-Extremal and Extremal Quantum-Corrected Two-Dimensional Charged Black Holes

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Near-extremal and extremal quantum-corrected two-dimensional charged black holes

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We consider charged black holes within dilaton gravity with exponential-linear dependence of action coefficients on dilaton and minimal coupling to quantum scalar fields. This includes, in particular, CGHS and RST black holes in the uncharged limit. For non-extremal configuration quantum correction to the total mass, Hawking temperature, electric potential and metric are found explicitly and shown to obey the first generalized law. We also demonstrate that quantum-corrected extremal black holes in these theories do exist and correspond to the classically forbidden region of parameters in the sense that the total mass $M_{tot} < Q$ (Q is a charge). We show that in the limit $T_H \rightarrow 0$ (where T_H is the Hawking temperature) the mass and geometry of non-extremal configuration go smoothly to those of the extremal one, except from the narrow near-horizon region. In the vicinity of the horizon the quantum-corrected geometry (however small quantum the coupling parameter κ would be) of a non-extremal configuration tends to not the quantum-corrected extremal one but to the special branch of solutions with the constant dilaton (2D analog of the Bertotti-Robinson metric) instead. Meanwhile, if $\kappa = 0$ exactly, the near-extremal configuration tends to the extremal one. We also consider the dilaton theory which corresponds classically to the spherically-symmetrical reduction from 4D case and show that for the

quantum-corrected extremal black hole $M_{tot} > Q$.

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I. INTRODUCTION

Two-dimensional (2D) models of dilaton gravity serve as an excellent tool which enables us to understand better the crucial points of black hole physics [1] in a more simple context than in the four-dimensional (4D) case (see for recent reviews [2], [3]). One of such issues is the role and character of backreaction caused by quantum fields. This backreaction reveals itself for both non-extremal black holes (NEBH) and extremal ones (EBH). In the first case it affects, in particular, the value of the Hawking temperature.

For the CGHS black hole [1] the corresponding correction was calculated in [4] for the Hartle-Hawking state and, in an independent approach, in [5] for the evaporating black hole. Further, the corresponding results were generalized to take into account the presence of a point-like shell that is necessary for the black hole to maintain thermal equilibrium with its Hawking radiation [9] (the influence of a shell on thermodynamics of 4D quantum-corrected black holes was considered in [10]). Quantum corrections to the geometry and thermodynamics of NEBH were also considered for another model which classically arises in the "spherically-symmetrical gravity" (SGG) theory as a result of spherically-symmetrical reduction from 4D case [6].

Consideration of [6] was generalized in [7] and [8] but the issue of relationship between quantum-corrected NEBH and EBH remained not completely clear in what concerns the limiting transition and comparison of physical quantities like mass and geometry. It is worth mentioning that in recent years the very existence of semiclassical EBH in general relativity became the subject of discussion [11], [12]. Direct treatment of quantum backreaction caused by massive fields on the Reissner-Nordström black hole [13] confirmed that the semiclassical EBH do exist in general relativity, with the extremality condition slightly modified due to quantum corrections. This conclusion was also extended to 2D generic dilaton gravity [14].

The aim of the present paper is to find in the explicit form quantum corrections to the metric and thermodynamic quantities of NEBH and trace in detail the extremal limit $T_H \rightarrow 0$ (T_H is the Hawking temperature) from quantum-corrected NEBH. We will see that in the near-horizon region the NEBH approaches not EBH but a special branch of solution with the constant dilaton field which is the 2D analogue of Bertotti-Robinson spacetime. We find quantum corrections to the mass of EBH and show that it agrees with the limiting value of NEBH. We also calculate quantum corrections to the mass of the EBH in the SGG model. It turns out that $M_{tot} < Q$ for the quantum-corrected CGHS charged black hole but $M_{tot} > Q$ for the latter case.

It is worth noting that in typical exactly solvable models always $T_H = const \neq 0$ [15]. Therefore, generic EBH (except some special classes with vanishing potentials [14] which have nothing to do with the models discussed in the present paper) do not fall into the family of exactly solvable models. In such a situation, one is led to using a perturbative approach with respect to the quantum-coupling parameter κ . In doing so, one should be careful about the choice of the unperturbed state since, as is aforementioned, the relationship between the mass and charge for the quantum-corrected EBH can be such that classical EBH with the same M_{tot} and Q are forbidden at all. It means that the problem should be considered in a self-consistent approach from the very beginning, with the behavior of the metric ensuring the condition $T_H = 0$.

To avoid complications which are unnecessary for the issues under consideration we discuss only backreaction caused by scalar field minimally coupled to the dilaton, so it is described by the Polyakov action. (The issue EBH in theories with the non-minimal coupling is the separate subject which is beyond the scope of the present paper. We only mention that rather recently the existence of EBH solutions was confirmed with the help of numeric calculations in [16].)

Throughout the paper we use the Schwarzschild gauge instead of that used for studying quantum corrections in [6] for SGG model and in [7] for a more general case. In doing so, we do not fix in advance the conformal frame, so all coefficients in the action (2) are generic

with the only constraint that classically the linear dilaton vacuum is the exact solution of field equations. To avoid complication connected with a finite size, in this paper we restrict ourselves to the system in an infinite space.

The paper is organized as follows. In Sec. 2 we list field equations for the quantum-corrected gravitation-dilaton system and suggest the simplified approach in which we find the general expression for the correction to T_H in terms of classical system directly from field equations. In Sec. 3 we apply the previous formulas to the charged version of CGHS black hole with modification of dilaton potentials which for the uncharged version ensured exact solvability. The explicit expressions for the metric, mass, temperature, potential and entropy are obtained in terms of the horizon value of the dilaton and charge are obtained and the validity of the first law is verified. In Sec. 4 we carry out independent consideration of semiclassical extremal black holes which from the very beginning fall into another topological sector. We find, how the relationship $M = Q$ between the mass and charge, typical of classical EBH, modifies due to quantum corrections. In Sec. 5 we discuss what happens to the metric of the quantum-corrected near-extremal NEBH in the immediate neighborhood of the horizon and show the qualitative difference here with the pure classical case. In Sec. 6 we repeat briefly calculations of the mass of EBH for the quantum-corrected SGG model. Sec. 7 contains summary and discussion of the results.

II. FIELD EQUATIONS

Let us consider the system governed by the action

$$I = I_{gd} + I_{PL} + I_q, \quad (1)$$

where gravitation-dilaton part

$$I_{gd} = \frac{1}{2\pi} \int_M d^2x \sqrt{-g} [F(\phi)R + V(\phi)(\nabla\phi)^2 + U(\phi)], \quad (2)$$

the contribution of the electromagnetic field is

$$I_q = -\frac{1}{4\pi} \int_M d^2x \sqrt{-g} W(\phi) F_{\mu\nu} F^{\mu\nu}, \quad (3)$$

$F_{\mu\nu} F^{\mu\nu} = 2F_{01}F^{01} \equiv -2E^2$ and we omit boundary terms that does not affect the form of field equations.

I_{PL} is the Polyakov-Liouville action [17] incorporating effects of Hawking radiation and its backreaction on spacetime for a multiplet of N minimal scalar fields. It is convenient to write it down in the form

$$I_{PL} = -\frac{\kappa}{2\pi} \int_M d^2x \sqrt{-g} \left[\frac{(\nabla\psi)^2}{2} + \psi R \right]. \quad (4)$$

Varying (4) with respect to ψ , it is easy to obtain that the function ψ obeys the equation

$$\square\psi = R, \quad (5)$$

where $\square = \nabla_\mu \nabla^\mu$, $\kappa = N/24$ is the quantum coupling parameter. Varying the action with respect to the metric, we get

$$T_{\mu\nu} \equiv T_{\mu\nu}^{(gd)} + T_{\mu\nu}^{(q)} + T_{\mu\nu}^{(PL)} = 0, \quad (6)$$

where

$$T_{\mu\nu}^{(gd)} = \frac{1}{2\pi} \{ 2(g_{\mu\nu} \square F - \nabla_\mu \nabla_\nu F) - U g_{\mu\nu} + 2V \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} V (\nabla\phi)^2 \}, \quad (7)$$

$$T_{\mu\nu}^{(PL)} = -\frac{\kappa}{2\pi} \{ \partial_\mu \psi \partial_\nu \psi - 2\nabla_\mu \nabla_\nu \psi + g_{\mu\nu} [2\square\psi - \frac{1}{2}(\nabla\psi)^2] \}, \quad (8)$$

$$T_{\mu\nu}^{(q)} = -\frac{W}{2\pi} (g_{\mu\nu} E^2 + 2g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}).$$

Variation of the action with respect to ϕ gives rise to the equation

$$RF' + U' + W'E^2 = 2V\square\phi + V'(\nabla\phi)^2, \quad (9)$$

where prime denotes derivative with respect to ϕ .

Let us write down the metric in the Schwarzschild gauge:

$$ds^2 = -f dt^2 + f^{-1} dx^2. \quad (10)$$

It is worth noting that field equations of the semiclassical system under discussion look very much like those for a pure classical one, but with coefficients, shifted according to (11)

$$\tilde{F} = F - \kappa\psi, \quad \tilde{V} = V - \kappa \frac{\psi'^2}{2}. \quad (11)$$

In this paper we will be dealing with static solutions only. This is possible due to the fact that the structure of field equations exhibits the existence of the Killing vector $\xi_\alpha = e_\alpha^\beta \mu_{;\beta}$, where $\mu' = \tilde{F}' \exp(-\int d\phi \frac{\tilde{V}}{\tilde{F}})$, which is assumed to be time-like for the static case. The latter means that the metric, the fields ψ and ϕ depend on the coordinate x only. Alternatively, one can use instead of x the dilaton field ϕ itself.

Let us consider a configuration with the fixed charge Q . It follows from Maxwell equations $(WF^{\mu\nu})_{;\nu} = 0$ that

$$F_{01} = -QW^{-1}\sqrt{-g} \quad (12)$$

Then field equations take the following explicit form:

$$2f \frac{d^2 \tilde{F}}{dx^2} + \frac{df}{dx} \frac{d\tilde{F}}{dx} - U_{eff} - \tilde{V} f \left(\frac{d\phi}{dx} \right)^2 = 0, \quad (13)$$

$$\frac{df}{dx} \frac{d\tilde{F}}{dx} - U_{eff} + \tilde{V} f \left(\frac{d\phi}{dx} \right)^2 = 0, \quad (14)$$

$$U_{eff} = U - Q^2 W^{-1}. \quad (15)$$

It is also convenient to take the difference of (13), (14) to get

$$\frac{d^2 \tilde{F}}{dx^2} = \tilde{V} \left(\frac{d\phi}{dx} \right)^2 \quad (16)$$

or

$$\left(\frac{d^2 F}{d\phi^2} - V \right) \left(\frac{d\phi}{dx} \right)^2 + \kappa \left[\frac{1}{2} \left(\frac{d\psi}{dx} \right)^2 - \frac{d^2 \psi}{dx^2} \right] + \frac{dF}{d\phi} \frac{d^2 \phi}{dx^2} = 0. \quad (17)$$

In the present paper, we will consider the models for which $\frac{d^2 F}{d\phi^2} - V = -\kappa \frac{c}{4}$, where c is a constant (the coefficient $\frac{1}{4}$ is singled out for convenience) and the linear dilaton vacuum $\phi = -z \equiv -\lambda x$ is the exact solution. This condition is not very restrictive and includes, for example, the most part of string-inspired models (see below).

Let us denote by "+"

 sign the quantities calculated on the horizon. Eq. (5) can be integrated for static metrics to give

$$\frac{d\psi}{dx} = f^{-1} \left[\left(\frac{df}{dx} \right)_+ - \frac{df}{dx} \right], \quad (18)$$

where the constant integration is chosen to ensure the regularity of ψ near the horizon of non-extreme black holes, where $f \sim x - x_+$.

Then in the first approximation, with the main correction taken into account,

$$\phi = -z + \kappa \rho, \quad (19)$$

$$\frac{dF}{d\phi} \frac{d^2 \rho}{dx^2} + \left[\frac{1}{2} \left(\frac{d\psi}{dx} \right)^2 - \frac{d^2 \psi}{dx^2} \right] = 0. \quad (20)$$

With the same accuracy,

$$\frac{dF}{d\phi} \frac{d^2 \rho}{d\phi^2} - s = \frac{c}{4}. \quad (21)$$

$$\frac{d\rho}{d\phi} = \int_{-\infty}^{\phi} d\phi \left(\frac{dF}{d\phi} \right)^{-1} \left(s + \frac{c}{4} \right). \quad (22)$$

$$s \equiv -f^{-1} \frac{d^2 f}{d\phi^2} + \frac{1}{2f^2} \left[\left(\frac{df}{d\phi} \right)^2 - \left(\frac{df}{d\phi} \right)_+^2 \right]. \quad (23)$$

At $x \rightarrow \infty$, $\phi \rightarrow -\infty$ we have a linear dilaton vacuum.

Another field equation is obtained as the sum of eqs. (13), (14):

$$f \frac{d^2 F}{dx^2} + \frac{df}{dx} \frac{dF}{dx} = U_{eff} - \kappa f''. \quad (24)$$

Rescaling the potential according to $U_{eff} = \lambda^2 u$, we have

$$u = \left(\frac{df}{d\phi} \frac{d\tilde{F}}{d\phi} + f \frac{d^2\tilde{F}}{d\phi^2} \right) \left(\frac{d\phi}{dz} \right)^2 + \frac{d\tilde{F}}{d\phi} \frac{d^2\phi}{dz^2} = \frac{d}{dz} \left(f \frac{d\tilde{F}}{dz} \right). \quad (25)$$

At the horizon $f = 0$ and we obtain directly from (24)

$$T_H = \frac{1}{4\pi} \left(\frac{df}{dx} \right)_+ = \frac{1}{4\pi} \left| \frac{U_{eff} - \kappa f''}{\frac{dF}{d\phi} \frac{d\phi}{dx}} \right|_+. \quad (26)$$

Retaining the terms of the first order in κ , we have $\frac{d\phi}{dz} = -(1 + \kappa \frac{d\rho}{d\phi}) + \dots$ and

$$T_H = \frac{\lambda}{4\pi} \left(\frac{\partial F}{\partial \phi} \right)_+^{-1} [u_+ - \kappa \left(\frac{d^2 f^{(0)}}{d\phi^2} + u_+ \left(\frac{d\rho}{d\phi} \right)_+], \quad (27)$$

where $f^{(0)}$ is the solution of classical field equations and $(\frac{\partial \rho}{\partial \phi})_+$ is given by the integral (22).

As, by assumption, classically

$$\frac{d^2 F}{d\phi^2} - V = 0, \quad (28)$$

it follows from field equations with $\kappa = 0$ that $\phi = -z$ and

$$f^{(0)} = \left(\frac{dF}{d\phi} \right)_+^{-1} \int_{\phi_+}^{\phi} d\phi u. \quad (29)$$

By substitution to eq. (27) we obtain

$$T_H = T_H^{(0)}(1 - \kappa D), \quad T_H^{(0)} = \frac{\lambda}{4\pi} \left(\frac{dF}{d\phi} \right)_+^{-1} u_+, \quad D = \frac{u'_+}{u_+ F'_+} - 2 \frac{F''_+}{F'^2_+} + \left(\frac{d\rho}{d\phi} \right)_+.$$

III. QUANTUM CORRECTIONS TO GEOMETRY AND THERMODYNAMICS

From now on we will consider the class of models with

$$F = \exp(-\phi) + \kappa \frac{b}{2} \phi, \quad V = \exp(-\phi) + \frac{\kappa}{4} c, \quad W = \exp(-\phi), \quad u = \exp(-\phi) - q^2 \exp(\phi), \quad (30)$$

where we rescaled the charge according to $Q = q\lambda$.

$$\tilde{F} = \exp(-\phi) + \kappa \frac{b+2}{2} \phi. \quad (31)$$

In the uncharged case $q = 0$ the system under consideration represents deformation of the CGHS model to which it reduces if $b = c = 0$. If $p \equiv b + \frac{c}{2} + 1 = 0$, we obtain the CN

model [18] that contains, as the particular cases, the RST model ($c = 0$, $b = -1$) [19] and BPP ($c = 2$, $b = -2$) [20]. In turn, the CN model is the particular class of a more wide family of exactly solvable models [15].

The classical limit $\kappa = 0$ of the model (30) was analyzed in [21], [22]. In this limit

$$f^{(0)} = 1 - 2me^\phi + q^2e^{2\phi} = 1 - \frac{2m}{r} + \frac{q^2}{r^2} = \left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right), \quad r \equiv \exp(-\phi). \quad (32)$$

The equation $f(r) = 0$ has two roots, if $m \geq q$,

$$r_{\pm} = \exp(-\phi_{\pm}) = m \pm \sqrt{m^2 - q^2}, \quad m = \frac{\exp(-\phi_+) + \exp(-\phi_-)}{2}, \quad q^2 = \exp[-(\phi_+ + \phi_-)], \quad (33)$$

the event horizon corresponding to r_+ . Hereafter, we denote by the index "+" the quantities calculated at r_+ . Then the Hawking temperature $T_H = \frac{1}{4\pi} \left(\frac{df}{dx}\right)_+$ is equal to

$$T_H^{(0)} = \frac{\lambda}{2\pi} \frac{\sqrt{m^2 - q^2}}{m + \sqrt{m^2 - q^2}}. \quad (34)$$

Now it is the issue of quantum corrections that we turn to. Calculating the quantity s in (23) with respect to the unperturbed metric, we obtain

$$s = \frac{1}{2} \left[a_0 + \frac{a_1 r_-}{r - r_-} + \frac{a_2 r_-^2}{(r - r_-)^2} \right]. \quad (35)$$

$$a_0 = -(1 - x)^2, \quad a_1 = 2(x + 1)(2 - x), \quad a_2 = 1 - x^2, \quad (36)$$

$$\frac{d\rho}{d\phi} = \frac{1}{2r} \left[(1 - x)^2 - \frac{c}{2} + (1 + x)\mu \right], \quad (37)$$

$$\mu = \frac{2 \ln(1 - y)}{y} + \frac{2 + y(x - 3)}{1 - y}, \quad (38)$$

where

$$x = \frac{r_-}{r_+}, \quad y = \frac{r_-}{r}.$$

Finding the solutions of eq. (25) perturbatively, we obtain

$$f = f^{(0)} + \kappa f^{(1)}, \quad (39)$$

$$f^{(0)} = (1 - y)(1 - u), \quad u \equiv \frac{r_+}{r}. \quad (40)$$

$$f_1 = \frac{\chi}{r}, \quad \chi = \chi_0 + \chi_1, \quad (41)$$

$$\chi_0 = \frac{(\phi - \phi_+)}{2} [(1 - x)^2 - \frac{c}{2}]. \quad (42)$$

$$\chi_1 = f^{(0)} \frac{p}{2} + \frac{(1 - u)}{2} [3y - 3 - 2(x + x^2) - \frac{c}{4}x(1 + u)] - 2(1 - u)(1 + x)(1 - y) \frac{\ln(1 - y)}{y} + \frac{G}{2}. \quad (43)$$

$$G = \frac{(1 + x)}{x} (1 - x)^2 \ln \left[\frac{1 - x}{1 - y} \right]. \quad (44)$$

In the afore-mentioned formulas the quantity the horizon "radius" r_+ is supposed to be fixed. For the quantum-corrected metric the geometry near r_- becomes in general singular (so r_- loses its direct meaning) in agreement with general properties of Cauchy horizons. Then, r_- should be understood as the formal quantity $r_- = \frac{q^2}{r_+}$.

A. Hawking temperature

Knowing the functions $f(\phi)$ (39)-(42) and $z(\phi)$ (19), (22) one can calculate in the same approximation the Hawking temperature which agrees with (27) for the model under discussion:

$$T_H = T_0 [1 - x + \kappa \eta \exp(\phi_+)], \quad T_0 \equiv \frac{\lambda}{4\pi}. \quad (45)$$

$$\eta = - \left[\frac{x^2 + x(1 + p) + 2 - p}{2} - \frac{(1 + x)(1 - x)}{x} \ln(1 - x) \right], \quad (46)$$

$$p = b + \frac{c}{2} + 1.$$

In the uncharged case $x = 0$, $\eta = \frac{1}{2}$ and we return to the result of [9]. If $\kappa \ll 1 - x$, the quantum correction is small as compared to the classical Hawking temperature. If $\kappa \sim 1 - x \ll 1$, the term with a logarithm can be neglected and the condition of the extremality changes to

$$1 - x = 2\kappa \exp(\phi_+). \quad (47)$$

It agrees with the general condition

$$U(\phi_+)F'(\phi_+) = \kappa U'(\phi_+) \quad (48)$$

which should hold on the event horizon of quantum-corrected extremal black holes (see [14], Sec. 8.1).

B. Potential

If we write $F_{01} = -\frac{\partial\varphi}{\partial\xi}$, then it follows from (12) that the potential

$$\varphi = -q \int dz \frac{1}{r} = q \int_{-\infty}^{\phi} d\phi (1 - \kappa\rho') \exp(\phi) = \frac{q}{r} + \varphi_1^{(1)}, \quad \varphi_1^{(1)} = \kappa q \varphi_1 \quad (49)$$

and after direct calculations we obtain

$$\varphi_1 = \frac{a}{2r_-^2}, \quad (50)$$

$$a(x) = x + x^3 + \frac{c}{4}x^2 + (1 - x^2) \ln(1 - x). \quad (51)$$

In the limit $q \rightarrow 0$, $r_- \rightarrow 0$, $x \rightarrow 0$ φ_1 does not contain q and we see that $\varphi_1^{(1)} \sim q \rightarrow 0$, as it should be.

C. Energy

The energy of the classical gravitational-dilaton system with the string-inspired Lagrangian was obtained in the closed form in [21], [22]. Its generalization to the generic quantum-corrected case is direct. The energy (see, for example, [7], [25]) is equal to

$$E = -\frac{1}{\pi} \left(\frac{d\tilde{F}}{dl} \right)_B = -\frac{1}{\pi} \left(\frac{d\tilde{F}}{dx} \sqrt{f} \right)_B = \frac{\lambda}{\pi} \varepsilon, \quad (52)$$

$$\varepsilon = \varepsilon^{(0)} + \kappa \varepsilon^{(1)}. \quad (53)$$

where f and \tilde{F} can be taken from eqs. (31), (39) - (42), dl is the proper distance. We are interested in the ADM mass $M_{tot} = \lim_{r \rightarrow \infty} (E - E_0)$, where E_0 is the energy of some background configuration.

Calculating (52), (53) and collecting all terms of the order κ together, we obtain at $r \rightarrow \infty$

$$\varepsilon^{(0)} = -r + \frac{r_+ + r_-}{2}. \quad (54)$$

$$\varepsilon_1 = -\frac{x}{2} + \frac{1}{4}(1-p) + \frac{cx}{16} - \frac{1}{4} \frac{(1+x)}{x} (1-x)^2 \ln(1-x) + \frac{(\phi_+ - \phi)}{4} (1-x)^2 + \frac{c(\phi - \phi_+)}{8} \quad (55)$$

D. Background

We choose, as the reference configuration, the "quasiflat" metric which would correspond in the limit $\kappa = 0$ to the classical solution $f = 1 + \frac{q^2}{r^2}$, $\phi = -z + const$ and represents at infinity the vacuum state with vanishing stresses. Formally, it means that we should replace in the expression (18) the quantity $(\frac{df}{dx})_+$ by zero. If $q = 0$, one can observe that the linear dilaton vacuum is the exact solution of one-loop field equations with $b = c = 0$ and all possible corrections come only due to non-zero b and c ("quasiflat" background). Repeating calculations with $q \neq 0$, we obtain from field equations (13), (14) that at $r \rightarrow \infty$

$$f = 1 + \frac{\kappa}{2r} (b + \frac{c}{2} - \frac{c}{2} \phi) + O(\frac{1}{r^3}),$$

the corresponding contribution to the energy

$$\varepsilon^{quasiflat} = -\frac{dF}{dz} \sqrt{f} = -r + \kappa \varepsilon_1 \quad (56)$$

$$\varepsilon_1^{quasi\flat} = -\frac{1}{2}\left(\frac{p-1}{2} - \frac{c\phi}{4}\right) - \frac{(1-p)}{2} \quad (57)$$

$$\varepsilon_1^{flat} = -\frac{1}{4}(1-p) + \frac{c\phi}{8} \quad (58)$$

$$\varepsilon_0 = \frac{r_+ + r_-}{2}. \quad (59)$$

$$\varepsilon_1 - \varepsilon_1^{flat} = \varepsilon^{th} - \frac{x}{2} - \frac{cx}{16} - \frac{1(1+x)}{4x}(1-x)^2 \ln(1-x) - \frac{c\phi_+}{8} + \frac{1-p}{2}, \quad (60)$$

$$\varepsilon^{th} = \frac{(\phi_+ - \phi)}{4}(1-x)^2, \quad (61)$$

$$M_{tot} = M_{th} + M_{BH}, \quad (62)$$

$$M_{th} = \kappa\lambda \frac{(\phi_+ - \phi)}{4\pi}(1-x)^2 = \frac{\pi}{6}T_H^2 L, \quad \lambda L = \phi_+ - \phi, \quad T_H = T_0(1-x), \quad T_0 = \frac{\lambda}{4\pi} \quad (63)$$

$$M_{BH} = M_{BH}^{(0)} + 2\kappa T_0 M_1, \quad M_{BH}^{(0)} = 2T_0(r_+ + r_-), \quad M_1 = -x + \frac{cx}{8} - \frac{1(1+x)}{2x}(1-x)^2 \ln(1-x) - \frac{c\phi_+}{4} \quad (64)$$

$$M_{tot} = M_{BH} + T_0\kappa(\phi_+ - \phi)(1-x)^2, \quad M_{BH} = 4T_0\left[\frac{r_+ + r_-}{2} + \kappa\left(m_1 - \frac{c\phi_+}{8}\right)\right] \quad (65)$$

$$m_1 = -\frac{x}{2} + c\frac{x}{16} - \frac{1(1+x)}{4x}(1-x)^2 \ln(1-x) \quad (66)$$

In the limit $x \rightarrow 0$ $m_1 = \frac{1}{4}$ in accordance with [9].

It follows from (62)-(65) that, independently of values of b and c the total mass splits to separate contributions M_{BH} and M_{th} which come from a black hole itself and thermal gas between a horizon having a temperature T_H .

E. Corrections to entropy

For completeness, we list also the explicit expressions for the entropy. It includes contribution S_0 from the horizon [21], [22] (2D analog of Bekenstein-Hawking entropy) and quantum corrections S_q [26], [27]. The constant in S_q is chosen to ensure $S_q \rightarrow 0$ when the boundary value $r_B \rightarrow r_+$ (no room for radiation).

$$S = S_0 + S_q, \quad S_0 = 2F(\phi_+).$$

$$S_q = 2\kappa(\psi_B - \psi_+) = -2\kappa \int_{\phi_+}^{\phi_B} \frac{[f_\phi^{(0)'} - f_+^{(0)'}]}{f} \quad (67)$$

$$S_q = \kappa s_q, \quad s_q = 2(\phi_+ - \phi_B)(1 - x) + 2(1 + x) \ln \frac{1 - x}{1 - y}. \quad (68)$$

In the limit $r_B \gg r_+$ we have

$$s_q = 2(\phi_+ - \phi_B)(1 - x) + 2(1 + x) \ln(1 - x). \quad (69)$$

The first term looks like the contribution of hot thermal gas in a flat space with the temperature T_H (with quantum correction omitted since s_q is itself multiplied by κ). The second term may be interpreted as the contribution from vacuum polarization and diverges in the extremal limit $x \rightarrow 0$.

F. First law

Having explicit expressions for the quantum-corrected mass, entropy, temperature and potential one can check by direct calculations that the general first law holds with quantum corrections taken into account:

$$\delta M_{tot} = T_H \delta S_{tot} - \varphi_+ \delta Q.$$

IV. QUANTUM-CORRECTED EXTREME CONFIGURATION

Up to now, we were dealing with non-extremal black holes. In this section we start from the very beginning from the configuration which belongs to another topological class - extremal black holes ($T_H = 0$), and find quantum corrections to the mass. Afterwards, we compare the result with what is obtained by the limiting transition from the NEBH to the EBH and show that the results coincide.

As is shown in [14], the metric describing a static black hole (non-extremal or extremal) can be represented in the form which includes only derivatives with respect to the dilaton (with the coordinate x removed):

$$f = \chi \exp(-\gamma), \quad \chi' = U \tilde{w} \exp(-\gamma), \quad (70)$$

$$\alpha = \frac{V - \frac{\kappa y^2}{2}}{w - \kappa y}, \quad (71)$$

$$\chi = \chi_0 - \kappa \chi_1, \quad \chi_0 = \int_{\phi_+}^{\phi} d\phi U \exp(-\gamma) w,$$

$$\chi_1 = \int_{\phi_+}^{\phi} d\phi U \exp(-\gamma) y$$

$$\gamma = \int d\phi \alpha, \quad \alpha \equiv \frac{\tilde{V}}{\tilde{w}}, \quad \tilde{w} = w - \kappa y, \quad y = \psi', \quad w = F'$$

From now on we put for simplicity $b = c = 0$. Then $w = -\exp(-\phi)$,

$$\alpha = -\frac{1 - \frac{\kappa y^2}{2} \exp(\phi)}{1 + \kappa y \exp(\phi)}. \quad (72)$$

In the pure classical case $\kappa = 0$ and eqs. (70)-(72) actually give us exact solutions in agreement with the known fact that classically any gravitation-dilaton 2D system is integrable. However, for $\kappa \neq 0$ we have non-linear integral-differential equations since f enters the right hand side via the quantity y . We may try to solve them approximately, putting in the right hand side $y = y^{(0)}$ in terms with the small parameter κ , where $y^{(0)}$

corresponds to the classical extremal solution ($\kappa = 0$) for a given value of ϕ_+ : $f^{(0)} = [1 - \exp(\phi - \phi_+)]^2$, so

$$y^{(0)} = \frac{2u}{1-u}, \quad u \equiv \exp(\phi - \phi_+)$$

$$U = \exp(-\phi)[1 - q^2 \exp(2\phi)].$$

The condition of extremality (48) gives us

$$\exp(\phi_+) = q + \kappa, \tag{73}$$

whence

$$U = \exp(-\phi)(1 - u^2) + 2\kappa u,$$

$$\alpha = \frac{V - \frac{\kappa y^2}{2}}{w - \kappa y},$$

$$\chi = \chi_0 - \kappa \chi_1, \quad \chi_0 = \int_{\phi_+}^{\phi} d\phi U \exp(-\gamma) w,$$

$$\chi_1 = \int_{\phi_+}^{\phi} d\phi U \exp(-\gamma) y,$$

$$\gamma = \int d\phi \alpha, \quad \alpha \equiv \frac{\tilde{V}}{\tilde{w}}, \quad \tilde{w} = w - \kappa y, \quad y = \psi', \quad w = F'.$$

We have in the approximation under consideration:

$$\gamma = - \int du T, \quad T = \frac{(1-u)^2 - 2\tilde{\kappa}u^3}{u(1-u)(1-u+2\tilde{\kappa}u^2)}, \quad \tilde{\kappa} \equiv \kappa \exp(\phi_+).$$

After integration, we obtain neglecting terms which contains κ^2 :

$$\exp(-\gamma) = \frac{(1-u)}{u_1 - u} u \exp(\phi_+) [1 - 2\tilde{\kappa} \ln(1-u)],$$

where $u_1 = 1 + 2\tilde{\kappa}$, the constant of integration is chosen to ensure $\gamma = -\phi$ in the classical limit $\kappa = 0$.

Then after simple but somewhat lengthy calculations we obtain

$$f = (1 - u)^2 + \tilde{\kappa}e(u) \quad (74)$$

$$e(u) = -4(1 - u)^2 \ln(1 - u) - u(1 - u)^2 \quad (75)$$

Now we can check the metric of the quantum-corrected EBH (74), (75) with the limiting form of the NEBH metric (39)-(42). In doing so, one should be careful since in (39)-(42) quantum corrections stem not only from the terms with χ , but also from the term $f^{(0)}$. This happens since it contains the parameter $r_- = \frac{q^2}{r_+}$ in which κ appears due to the extremality condition (73) relating q and r_+ . With this in mind, one can check that the results of implementing the extremality conditions to (39)-(42) do coincide with eqs. (74)-(75). Thus, the metric of the non-extremal configuration tends to that of the extremal one in the limit under discussion.

For small u

$$f = 1 - B \exp(\phi - \phi_+) \equiv 1 - Bu, \quad B = 2 - 3\tilde{\kappa}.$$

As is shown in [14], it follows from field equations that

$$\frac{\partial \phi}{\partial z} = z_0 \tilde{w}^{-1} e^\gamma, \quad z_0 = \text{const.} \quad (76)$$

In the main approximation

$$\frac{d\phi}{dz} = -\frac{B}{1 + \kappa y \exp(\phi)} \frac{(u_1 - u)}{1 - u} [1 + 2\tilde{\kappa} \ln(1 - u)]$$

At infinity the quantum corrections vanish and we have the linear dilaton vacuum: $\frac{\partial \phi}{\partial z} = -1$, whence

$$B = \frac{1}{u_1} = 1 - 2\tilde{\kappa}$$

and

$$\frac{d\phi}{dz} = -1 + \tilde{\kappa}O(u^2), \quad u \ll 1.$$

Applying now eq. (52), one can calculate the energy at infinity, knowing expansions for $f(\phi)$ and $\frac{\partial\phi}{\partial z}$ and subtracting the energy of the corresponding quasiflat configuration. Calculations go along the same line as in the non-extremal case and give us

$$m = \exp(-\phi_+) - \frac{3}{2}\kappa.$$

With the same accuracy

$$m = q - \frac{\kappa}{2}, \quad m < q.$$

Comparing the expression for the energy of the near extremal BH, taking into account the extremality condition and neglecting terms κ^2 , we see that

$$m = \frac{\exp(-\phi_+) + q^2 \exp(\phi_+)}{2} - \frac{\kappa}{2} \rightarrow \exp(-\phi_+) - \frac{3}{2}\kappa.$$

Thus, the mass of the quantum-corrected non-extremal configuration tends to that of the pure extreme one when the relationship between q and ϕ_+ approaches the extremality condition (48). It is worth noting that the account for backreaction of massive fields in general relativity also gives the result that for the quantum-corrected extremal charge black hole $m < q$ [13].

Thus, the relationship between mass and charge of the extremal configuration is situated in the space of parameters in the region where the very existence of black holes is forbidden classically.

V. EXTREME LIMIT OF QUANTUM-CORRECTED NON-EXTREME CONFIGURATION

The formulas for the quantum-corrected metric (74), (75) fail in the immediate vicinity of the horizon. Indeed, in this vicinity $y \rightarrow \infty$, while the coefficients $V(\phi_+)$, $w(\phi_+)$ are finite. Therefore the terms of the type κy or κy^2 not only cannot be considered in eq. (71) as small corrections but actually dominates the corresponding expressions. The asymptotic

form of the metric of a generic quantum-corrected extremal black hole was found in [14] that generalized the previous result of [23] for the particular model. It turned out that

$$f \sim (\phi - \phi_+)^{\varepsilon}, \quad (77)$$

where for small κ the quantity $\varepsilon = 2 + \kappa\nu$, the coefficient ν (whose exact form is irrelevant for us now) depends on the behavior of V and F near the horizon. It is essential that the dependence of the metric on dilaton and that of dilaton on the Schwarzschild coordinate z is non-analytical in this region. On the other hand, it is obvious that for the non-extremal configuration, even if it is close to the extremal one as nearly as one likes, the metric is analytical there. How it may happen and what is going with the non-extremal metric in the extremal limit?

Let us trace the transition under discussion in more detail. For any non-extremal configuration we may exploit the power expansion near the horizon:

$$f = a_1(z - z_+) + a_2(z - z_+)^2 + a_3(z - z_+)^3 + \dots \quad (78)$$

$$\phi = \phi_+ + b_1(z - z_+) + b_2(z - z_+)^2 + b_3(z - z_+)^3 + \dots \quad (79)$$

It is convenient to use eqs. (24) and the dilaton equation which can be obtained by variation with respect to ϕ :

$$RF' + U' = 2V\Box\phi + V'(\nabla\phi)^2. \quad (80)$$

Substituting into them (41) and (79), taking into account that $R = -f''$ and equating coefficients at equal powers of $z - z_+$, we obtain the relations between coefficients. In the limit $a_1 \rightarrow 0$ (extreme limit) they simplify to give

$$U'(\phi_+) = 2a_2F'(\phi_+), \quad (81)$$

$$b_1[U'(\phi_+) - 2a_2F'(\phi_+)] = 6\kappa a_3, \quad (82)$$

whence we obtain

$$\kappa a_3 = 0. \tag{83}$$

For the classical system, when $\kappa = 0$ exactly, this turns into identity and does not impose any restriction on a_3 . Continuing the procedure order by order, one may restore the whole expansion which can be also obtained directly from the exact classical expression for the extremal black hole $f^{(0)} = [1 - \exp(\phi - \phi_+)]^2$. However, for the quantum-corrected case the situation is qualitatively different. Whatever small κ be, it entails immediately $a_3 = 0$. Continuing the procedure, one can easily obtain that all $a_n = 0$ with $n \geq 3$. In a similar way, all $b_n = 0$.

Let me remind that, apart from the main branch of solution, $\phi(z)$, for the gravitation-dilaton system there also exists the special one f_c with $\phi = \phi_0 = \text{const}$ (2D analogs of Bertotti-Robinson spacetime). For exactly solvable models the values $\phi_0 = \phi_s$ correspond to the singularity of the main branch, [15], [24], while in the generic case ϕ_s , [14], [28] coincides with the value typical of the extremal horizon. This second branch manifests itself just now.

Thus, in the vicinity of the horizon ($z \rightarrow 0$) in the extremal limit ($T_H \rightarrow 0$)

$$f_n(z, \kappa, T_H) \rightarrow f_c(z, \kappa)$$

but not to the quantum-corrected extremal solution f_e .

Meanwhile, for the pure classical system

$$f_n(z, 0, T_H) \rightarrow f_e(z, 0) \tag{84}$$

smoothly for all z .

Beyond the immediate vicinity of the horizon the situation is qualitatively different: as we saw, (84) holds true.

The size of this vicinity is governed by the parameter $k = \kappa |\ln(\phi - \phi_+)|$. If $k \ll 1$, the additional factor $(\phi - \phi_+)^{\kappa\nu}$ in the metric function due to quantum corrections is close to 1 and the relationship between the geometry of NEBH and EBH is similar to that for the pure classical case. However, if $k \gtrsim 1$, the quantum corrections changes the picture significantly

and should be taken into account from the very beginning that just leads to the expression like (77).

VI. SPHERICALLY-SYMMETRICAL REDUCTION AND QUANTUM CORRECTIONS

In this section we consider another physically relevant 2D model of dilaton gravity which classically appears within the framework of SSG. As in the previous sections, it is assumed that the quantum backreaction is due to minimal fields, so it is described by the Polyakov-Liouville action (4). Quantum corrections for such a system were studied in [6] for NEBH. Therefore, we restrict ourselves by the case of EBH only. Now the action looks like (2) with coefficients

$$F = r^2, w = 2\phi, V = 2, U = 2\left(1 - \frac{Q^2}{r^2}\right), r = \phi,$$

and the common factor $\frac{1}{4}$ instead of $\frac{1}{2\pi}$ (see [6] for details). As calculations run almost along the same lines as for the model discussed in the text, above, I list basic formulas and results only.

For the classical extremal black hole $f_{ext}^{(0)} = \left(1 - \frac{r_+}{r}\right)^2$, so $y^{(0)} = \frac{2r_+}{r^2\left(1 - \frac{r_+}{r}\right)}$. The extremality condition (48) gives now

$$Q = r_+ - \frac{\kappa}{2r_+}. \tag{85}$$

The quantum-corrected metric function has the form (74), where now

$$\epsilon(u) = -4 + 5u - u^2 - \frac{4(1-u)^2}{u} \ln(1-u).$$

It follows from (76) that at large distances $\frac{\partial\phi}{\partial r} = 1 + \kappa O\left(\frac{1}{r^3}\right)$. Then, calculating the energy at infinity we obtain after some algebra that

$$M_{tot} = E - E_0 = r_+ + \frac{\kappa}{2r_+},$$

where E_0 is the contribution of the flat reference background. Taking into account the extremality condition (85) we obtain

$$M_{tot} = Q + \frac{\kappa}{r_+} > Q.$$

Thus, in contrast to the charged CGSH black hole, now the extremal quantum-corrected black hole lies inside the classically allowed region of parameters.

VII. SUMMARY

We have found quantum corrections caused by minimal fields to the geometry and thermodynamics of non-extremal charged black holes in the Hartle-Hawking state for string-inspired dilaton theories of gravity that include in the uncharged case CGHS black holes. In the limit of the zero charge the quantum corrections for the non-extremal black hole agree with our previous calculations [9]. The validity of the first general law is demonstrated with quantum corrections taken into account.

We have shown that the mass splits in two pieces. The first one (M_{BH}) is the mass of the black itself that contains the horizon dilaton value ϕ_+ but does not depend on the dilaton field in the point of observation in an asymptotically flat region ϕ_B . The second one (M_{th}) is proportional to the difference $\phi_+ - \phi_B$ and coincides with that of a thermal gas at the temperature T_H in a box of corresponding size in the flat space. In the extremal limit $M_{th} \rightarrow 0$, while M_{BH} remains finite. Meanwhile, the quantum correction to the entropy (69) diverges in the extremal limit that agrees with previous observations [7], [8] where it was attributed to the failure of the one-loop approximation. However, in our view, this does not mean necessarily that such failure happens. Rather, it looks like the consequence of the changes in geometry: the bulk contribution from S_q becomes divergent because of the fact that the proper distance between the horizon and any other point tends to infinity in the extremal limit. We also obtained, in contrast to [7], the finite shift in the horizon value of the dilaton for EBH that is described for a given charge by very simple

formulas (73), (85). The reason of discrepancy is not quite clear. We only suppose that it is somehow connected with the different definitions of the unperturbed state in the situation of competition between corrections of two kinds connected with quantum effects and small deviation from the extremality.

The results of the present paper along with the previous one [14] confirm the existence of semiclassical 2D EBH black holes for massless minimal fields. We traced the limiting transition from the quantum-corrected non-extremal black hole to the extremal one and showed that far from the horizon the geometry and mass of the first type configuration tend smoothly to those of the second one. However, in the vicinity of the horizon the situation changes: in the limit under discussion the solution tends to that with the constant dilaton that represents the 2D analogue of the Bertotti-Robinson spacetime. Meanwhile, for the pure classical black hole the limiting transition is uniform in the sense that non-extremal configurations go smoothly to the extremal one everywhere.. Thus, there is a crucial difference in this respect between classical and quantum-corrected black hole configurations. This phenomenon, along with the non-analytical behavior of the extremal metric on the Schwarzschild coordinate [23] show that quantum backreaction becomes crucial for properties of extremal black holes. Meanwhile, it does not affect their very existence but only changes slightly the condition of extremality.

Calculation of the mass of the quantum-corrected EBH showed that, depending on the model, $M_{tot} < Q$ or $M_{tot} > Q$. It is worth mentioning that $M_{tot} < Q$ for backreaction of massive fields on 4D quantum-corrected extremal Reissner-Nordström black hole [13].

Thus, in the simplified 2D context we manage to trace how quantum backreaction change properties of near-extremal and extremal black holes.

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- [1] C.G. Callan, S. Giddings, J.A. Harvey, and A. Strominger, *Phys. Rev. D* 45 (1992) R1005.
- [2] S.Nojiri and S. Odintsov, *Int. J. Mod. Phys. A* 16 (2001) 1015.
- [3] D. Grumiller, W. Kummer and D. V. Vassilevich, *Phys.Rept.* 369 (2002) 327.
- [4] O.B.Zaslavskii, *Phys.Lett.B* 375 (1996) 43.
- [5] D. Grumiller, W. Kummer, and D.V. Vassilevich, *JHEP* 0307 (2003) 009.
- [6] V.P.Frolov, W. Israel, and S.N.Solodukhin, *Phys.Rev. D*54 (1996) 2732.
- [7] A. J. Medved and G. Kunstatter, *Phys. Rev. D* 60 (1999) 104029.
- [8] M. Burić and V. Radovanović, *Class. Quant. Grav.* 16 (1999) 3937.
- [9] O.B. Zaslavskii, *Phys. Rev. D* 69 (2004) 044008.
- [10] O. B. Zaslavskii, *Phys. Rev. D* 68 (2003) 127502.
- [11] D. A. Lowe, *Phys. Rev. Lett.* 87, 029001 (2001).
- [12] P. R. Anderson, W. A. Hiscock, B.Taylor, Zero temperature black holes in semiclassical gravity, gr-qc/0102111.
- [13] J. Matyjasek and O. B. Zaslavskii, *Phys. Rev. D* 64 (2001) 104018.
- [14] O. B. Zaslavskii, *Class. Quant. Grav.* 20 (2003) 137.
- [15] O. B. Zaslavskii, *Phys. Rev. D* 59 (1999) 084013.
- [16] C. Barbachoux and A. Fabbri, *Phys. Rev.* 66 (2002) 024012.
- [17] A. M. Polyakov, *Phys. Lett. B* 103 (1981) 207.

- [18] J. Cruz and J. Navarro-Salas, Phys. Lett. B 375 (1996) 47.
- [19] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. D 46 (1992) 3444; Phys. Rev. D 47 (1992) 533.
- [20] S. Bose, L. Parker, and Y. Peleg, Phys. Rev. D 52 (1995) 3512.
- [21] V. P. Frolov, Phys. Rev. D 46 (1992) 5383.
- [22] G.W. Gibbons and M.J. Perry Int.J.Mod.Phys. D1 (1992) 335.
- [23] S. Trivedi, Phys. Rev. D 47 (1993) 4233.
- [24] S. N. Solodukhin, Phys. Rev. D 53 (1996) 824.
- [25] O.B. Zaslavskii, Foundations of Physics 33 (2003) 1.
- [26] R. C. Myers, Phys. Rev. D 50 (1994) 6412.
- [27] D. V. Fursaev and S. N. Solodukhin, Phys. Rev. D 52 (1995) 2133.
- [28] J. Cruz, A. Fabbri, D. J. Navarro and J. Navarro-Salas, Phys. Rev. D 61 (1999) 024011.