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LEPTOGENESIS AS THE ORIGIN OF MATTER

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Abstract

We explore in some detail the hypothesis that the generation of a primordial lepton-antilepton asymmetry (Leptogenesis) early on in the history of the Universe is the root cause for the origin of matter. After explaining the theoretical conditions for producing a matter-antimatter asymmetry in the Universe we detail how, through sphaleron processes, it is possible to transmute a lepton asymmetry – or, more precisely, a (B-L)-asymmetry – into a baryon asymmetry. Because Leptogenesis depends in detail on properties of the neutrino spectrum, we review briefly existing experimental information on neutrinos as well as the seesaw mechanism, which offers a theoretical understanding of why neutrinos are so light. The bulk of the review is devoted to a discussion of thermal Leptogenesis and we show that for the neutrino spectrum suggested by oscillation experiments one obtains the observed value for the baryon to photon density ratio in the Universe, independently of any initial boundary conditions. In the latter part of the review we consider how well Leptogenesis fits with particle physics models of dark matter. Although axionic dark matter and Leptogenesis can be very naturally linked, there is a potential clash between Leptogenesis and models of supersymmetric dark matter because the high temperature needed for Leptogenesis leads to an overproduction of gravitinos, which alter the standard predictions of Big Bang Nucleosynthesis. This problem can be resolved, but it constrains the supersymmetric spectrum at low energies and the nature of the lightest supersymmetric particle (LSP). Finally, as an illustration of possible other options for the origin of matter, we discuss the possibility that Leptogenesis may occur as a result of non-thermal processes.

Contents

1 Introduction

Our understanding of the Universe has deepened considerably in the last 25 years, so much so that a standard cosmological model has emerged [1]. In this model, after the Big Bang, a period of inflationary expansion [2] ensued that effectively set the Universe's curvature to zero. After inflation the Universe's expansion continued, not in an exponential fashion but with the rate of expansion being determined by which component of the Universe's energy density dominated the total energy density.

In the present epoch this energy density is dominated by a, so-called, dark energy component whose negative pressure causes the Universe's expansion to accelerate [3]. Dark energy now accounts for approximately 70% of the total energy density, with the other 30% of the remaining energy density of the Universe's being dominated by some kind of non-luminous (dark) matter. In detail, the angular distribution of the temperature fluctuations of the microwave background radiation measured by the Wilkinson Microwave Anisotropy Probe (WMAP) collaboration [4] determines the various components of the ratio of the Universe's energy density now ρ_o to the critical energy density ρ_c, $\Omega = \rho_o/\rho_c$.[1] The results are: $\Omega_{\rm dark\ energy} = 0.73 \pm 0.04$; $\Omega_{\rm matter} = 0.27 \pm 0.04$; and $\Omega_{\rm B} = 0.044 \pm 0.004$. Here $\Omega_{\rm B}$ is the contribution of baryonic matter to $\Omega_{\rm matter}$, confirming that about 85 % of $\Omega_{\rm matter}$ is indeed contributed by dark matter. The contribution of neutrinos and photons is at a few per mil, or below, and is negligible.

Although the standard cosmological model sketched above provides an accurate description of the present Universe and its evolution, deep questions remain to be answered. What exactly constitutes the dark energy? Is it just a cosmological constant? But if that is so, why is the energy scale associated with the corresponding vacuum energy density $[\rho_{\rm cc} = E_o^4; E_o \simeq 2 \times 10^{-3}$ eV$]$ so small? Equally mysterious is the nature of the dark matter, although in this case there are at least some particle physics candidates that may be the source for this component of the Universe's energy density.

[1] The critical density ρ_c is the density that corresponds to a closed Universe now, $\rho_c = 3H_o^2/8\pi G_N$. Here H_o is the value of the Hubble parameter now. Inflation predicts that $\rho_o = \rho_c$, so that $\Omega = 1$.

A further mystery is associated with the observed baryon energy density. This number can be used to infer the ratio of the number density of baryons to photons in the Universe, a quantity that is measured independently from the primordial nucleosynthesis of light elements. The WMAP results [4] are in agreement with the most recent nucleosynthesis analysis of the primordial Deuterium abundance, but there are discrepancies with both the inferred ^{4}He and ^{7}Li values [5]. These latter values, however, may have an underestimated error [6]. Averaging the WMAP result only with that coming from the primordial abundance of Deuterium gives:

$$\frac{n_B}{n_\gamma} \equiv \eta_B = 6.1 \pm 0.3 \times 10^{-10}. \tag{1}$$

Why does this ratio have this value?

In this review, we will principally try to address this last question which, as we shall see, is intimately related to the existence of a primordial matter-antimatter asymmetry. Nevertheless, we shall try, when germane, to connect our discussion with the broader issues of what constitutes dark energy and dark matter.

There is good evidence that the Universe is mostly made up of matter, although it is possible that small amounts of antimatter exist [7]. However, antimatter certainly does not constitute one of the dominant components of the Universe's energy density. Indeed, as Cohen, de Rujula, and Glashow [8] have compellingly argued, if there were to exist large areas of antimatter in the Universe they could only be at a cosmic distance scale from us. Thus, along with the question of why n_B/n_γ has the value given in Eq. (1), there is a parallel question of why the Universe is predominantly composed of baryons rather than antibaryons.

In fact, these two questions are interrelated. If the Universe had been matter-antimatter symmetric at temperatures of O(1 GeV), as the Universe cools further and the inverse process $2\gamma \to B + \bar{B}$ becomes ineffective because of the Boltzmann factor, the number density of baryons and antibaryons relative to photons would have been reduced dramatically as a result of the annihilation process $B + \bar{B} \to 2\gamma$. A straightforward calculation gives, in this case, [9]:

$$\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \simeq 10^{-18}. \tag{2}$$

Thus, in a symmetric Universe the question is really why observationally n_B/n_γ is so large!

It is very difficult to imagine processes at temperatures below a GeV that could enhance the ratio of the number density of baryons relative to that of photons much beyond the value this quantity attains when baryon-antibaryon annihilation occurs.[2]

[2] An exception is provided by some versions of Affleck-Dine Baryogenesis [10] where a baryon excess

Thus, because Eq. (2) does not agree with the observed value given in Eq. (1), one is led to the interesting conclusion that a primordial matter-antimatter asymmetry must have existed at temperatures of $\mathcal{O}(1 \text{ GeV})$ in the Universe. The observed value for n_B/n_γ and the lack of antimatter in the Universe are manifestations of this primordial asymmetry. Hence, in reality, the ratio η_B is, in effect, a measure of the number density of matter minus that of antimatter relative to the photon number density:

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = 6.1 \pm 0.3 \times 10^{-10}. \tag{3}$$

It is interesting to consider the physical origins of this primordial matter-antimatter asymmetry. From the seminal work of Sakharov [11] one knows that, under certain conditions which we will amplify later on in this article, this asymmetry can be generated by physical processes. In this review we will focus on Leptogenesis – the creation of a primordial lepton-antilepton asymmetry – as the root source for the observed baryon-antibaryon asymmetry of Eq. (3) [12]. In our view, Leptogenesis provides the most compelling scenario for generating the observed baryon asymmetry in the Universe. In particular, because Leptogenesis is closely linked with parameters in the neutrino sector that can be eventually determined experimentally, this scenario can be tested and can be either confirmed or ruled out by data.

The plan of this review is as follows. In Section 2 we discuss the theoretical conditions necessary for producing a primordial matter- antimatter asymmetry in the Universe and explain how, through a mechanism first discussed by Kuzmin, Rubakov, and Shaposhnikov, [13], it is possible to turn a lepton asymmetry into a baryon asymmetry. In Section 3 we review existing experimental information on the neutrino sector, as well as the seesaw mechanism [14] that provides a theoretical framework for understanding this data. Section 4 discusses thermal Leptogenesis and contains the main quantitative results of the review. In particular, we show in this Section that the observed value for η_B obtained from Leptogenesis significantly constrains low energy neutrino properties, and vice versa. In Section 5 we turn to dark matter and discuss how supersymmetric candidates for dark matter are significantly constrained if thermal Leptogenesis is the source of the observed baryon asymmetry in the Universe. The constraint arises from the overproduction of gravitinos. Section 6 discusses nonthermal Leptogenesis and other nonthermal processes that can lead to Baryogenesis. Finally, we present our conclusion and summary of results in Section 7.

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is produced by the decay of a scalar field very late in the history of the Universe, which reheats the Universe to temperatures of the $\mathcal{O}(100 \text{ MeV})$.

2 Theoretical Foundations

2.1 Sakharov's Conditions for Baryogenesis

In 1967, Sakharov [11] considered the consequences of the hypothesis that the observed expanding Universe originated from a superdense initial state with temperature of order the Planck mass, $T_i \sim M_{\rm P}$. Since he could not imagine how, starting from such an initial state, one could obtain a macroscopic separation of matter and antimatter, he concluded that our Universe contains today only matter. That is, the Universe evolved from an initial state even under charge conjugation to a state odd under charge conjugation today. He then realized that in an expanding Universe a matter-antimatter asymmetry could be generated dynamically, if C, CP, and baryon and lepton number were violated, and these processes were out of thermal equilibrium.

Sakharov also described a concrete model for Baryogenesis. He proposed as the origin for the baryon and lepton asymmetry the CP-violating decays of maximons, hypothetical neutral spin zero particles with mass of order the Planck mass. Their existence leads to a departure from thermal equilibrium already at temperatures $T \sim M_{\rm P}$, where a small matter-antimatter asymmetry is then generated. An unavoidable consequence of this model is that protons are unstable and decay. However, the proton lifetime in Sakharov's model turned out to be unobservably long, $\tau_p > 10^{50}$ years.

During the past four decades many models of Baryogenesis have been proposed, demonstrating that the conditions Sakharov spelled out to allow Baryogenesis to take place are quite readily satisfied in the Standard Model of particle physics and its extensions. There are, however, significant differences among the various mechanisms suggested for producing the baryon asymmetry. Grand Unified Theories (GUTs) have been of particular importance for the development of realistic models of Baryogenesis [15]. These theories provide natural heavy particle candidates, whose decays can be the source of the baryon asymmetry. However, in general, the simplest GUT models based on $SU(5)$ lead to a creation of a (B+L)-asymmetry, with a vanishing asymmetry for B-L. As will be made clear below, a (B+L)-asymmetry generated at the GUT scale eventually gets erased by sphaleron processes. In Leptogenesis, heavy Majorana neutrinos required by the seesaw mechanism [14] serve to trigger Baryogenesis. Because B-L is violated, the erasure present in GUTs is avoided. In principle, Electroweak Baryogenesis [16] is also an attractive possibility, as the relevant parameters could then be tested in collider experiments. However, in general, the electroweak phase transition is not sufficiently out of equilibrium to generate an asymmetry of the magnitude observed in the Universe. Finally, in supersymmetric theories the baryon and lepton number stored in scalar expectation values can also lead to Baryogenesis, through the, so-called, Affleck-Dine mechanism [10], which will be discussed in some detail in Section 6.

2.2 $B+L$ Violation in the Standard Model

Due to the chiral nature of the electroweak interactions, baryon and lepton number are not conserved in the Standard Model [17]. The divergence of the B and L currents,

$$J_\mu^B = \frac{1}{3} \sum_{generations} \left(\overline{q_L}\gamma_\mu q_L + \overline{u_R}\gamma_\mu u_R + \overline{d_R}\gamma_\mu d_R\right) , \tag{4}$$

$$J_\mu^L = \sum_{generations} \left(\overline{l_L}\gamma_\mu l_L + \overline{e_R}\gamma_\mu e_R\right) , \tag{5}$$

is given by the triangle anomaly,

$$\begin{aligned} \partial^\mu J_\mu^B &= \partial^\mu J_\mu^L \\ &= \frac{N_f}{32\pi^2}\left(-g^2 W_{\mu\nu}^I \widetilde{W}^{I\mu\nu} + g'^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}\right) . \end{aligned} \tag{6}$$

Here N_f is the number of generations, and W_μ^I and B_μ are, respectively, the $SU(2)$ and $U(1)$ gauge fields with gauge couplings g and g'.

As a consequence of the anomaly, the change in baryon and lepton number is related to the change in the topological charge of the gauge field,

$$\begin{aligned} B(t_f) - B(t_i) &= \int_{t_i}^{t_f} dt \int d^3x \partial^\mu J_\mu^B \\ &= N_f[N_{cs}(t_f) - N_{cs}(t_i)] , \end{aligned} \tag{7}$$

where

$$N_{cs}(t) = \frac{g^3}{96\pi^2}\int d^3x \epsilon_{ijk}\epsilon^{IJK} W^{Ii} W^{Jj} W^{Kk} . \tag{8}$$

For vacuum to vacuum transitions W^{Ii} is a pure gauge configuration and the Chern-Simons numbers $N_{cs}(t_i)$ and $N_{cs}(t_f)$ are integers.

In a non-abelian gauge theory there are infinitly many degenerate ground states, which differ in their value of the Chern-Simons number, $\Delta N_{cs} = \pm 1, \pm 2, \ldots$. The corresponding points in field space are separated by a potential barrier whose height is given by the so-called sphaleron energy E_{sph} [18]. Because of the anomaly, jumps in the Chern-Simons number are associated with changes of baryon and lepton number,

$$\Delta B = \Delta L = N_f \Delta N_{cs} . \tag{9}$$

Obviously, in the Standard Model the smallest jump is $\Delta B = \Delta L = \pm 3$.

In the semiclassical approximation, the probability of tunneling between neighboring vacua is determined by instanton configurations. In the Standard Model, $SU(2)$ instantons lead to an effective 12-fermion interaction

$$O_{B+L} = \prod_{i=1\ldots 3} \left(q_{Li}q_{Li}q_{Li}l_{Li}\right) , \tag{10}$$

which describes processes with $\Delta B = \Delta L = 3$, such as

$$u^c + d^c + c^c \to d + 2s + 2b + t + \nu_e + \nu_\mu + \nu_\tau \; . \tag{11}$$

The transition rate is determined by the instanton action and one finds [17]

$$\begin{aligned} \Gamma &\sim e^{-S_{\rm inst}} = e^{-\frac{4\pi}{\alpha}} \\ &= \mathcal{O}\left(10^{-165}\right) \; . \end{aligned} \tag{12}$$

Because this rate is extremely small, $(B+L)$-violating interactions appear to be completely negligible in the Standard Model. However, this picture changes dramatically when one is in a thermal bath.

2.3 Sphalerons and the KRS Mechanism

As emphasized in the seminal paper of Kuzmin, Rubakov and Shaposhnikov [13], in the thermal bath provided by the expanding Universe one can make transitions between the gauge vacua not by tunneling, but through thermal fluctuations over the barrier. For temperatures larger than the height of the barrier, the exponential suppresion in the rate provided by the Boltzmann factor disappears completely. Hence (B+L)-violating processes can occur at a significant rate and these processes can be in equilibrium in the expanding Universe.

The finite-temperature transition rate in the electroweak theory is determined by the sphaleron configuration [18], a saddle point of the field energy of the gauge-Higgs system. Fluctuations around this saddle point have one negative eigenvalue, which allows one to extract the transition rate. The sphaleron energy is proportional to $v_F(T)$, the finite-temperature expectation value of the Higgs field, and one finds

$$E_{sph}(T) \simeq \frac{8\pi}{g} v_F(T) \; . \tag{13}$$

Taking translational and rotational zero-modes into account, one obtains for the transition rate per unit volume in the Higgs phase [19]

$$\frac{\Gamma_{B+L}}{V} = \kappa \frac{M_W^7}{(\alpha T)^3} e^{-\beta E_{sph}(T)} \; , \tag{14}$$

where $\beta = 1/T$, $M_W = g^2 v_F(T)/2$ and κ is some constant.

Extrapolating this semiclassical formula to the high-temperature symmetric phase, where $v_F(T) = 0$, and using for M_W the thermal mass, $M_W \sim g^2 T$, one expects in this phase $\Gamma_{B+L}/V \sim (\alpha T)^4$. However, detailed studies have shown that this naive extrapolation from the Higgs to the symmetric phase is not quite correct. The relevant spatial scale for non-perturbative fluctuations is the magnetic screening length $\sim 1/(g^2T)$, but the corresponding time scale turns out to be $1/(g^4T\ln g^{-1})$, which is larger for small coupling [20, 21]. As a consequence one obtains for the sphaleron rate in the symmetric phase

$$\Gamma_{B+L}/V \sim \alpha^5 \ln \alpha^{-1} T^4. \tag{15}$$

It turns out that the dynamics of low-frequency gauge fields can be described by a remarkably simple effective theory, derived by Bödeker [21]. The color magnetic and electric fields satisfy the equation of motion

$$\vec{D} \times \vec{B} = \sigma \vec{E} - \vec{\zeta}\ . \tag{16}$$

Here $\vec{\zeta}$ is Gaussian noise, a random vector field with variance

$$\langle \zeta_i(x) \zeta_j(x') \rangle = 2\sigma \delta_{ij} \delta^4(x - x')\ . \tag{17}$$

These equations define a stochastic three-dimensional gauge theory. The parameter σ is the 'color conductivity', $\sigma = m_D^2/(3\gamma)$, where $m_D \sim gT$ is the Debye screening mass and $\gamma \sim g^2 T \ln(1/g)$ is the hard gauge boson damping rate. To leading-log accuracy one has $1/\sigma \sim \ln g^{-1}$. A next-to-leading order analysis yields for the sphaleron rate [22]

$$\frac{\Gamma_{B+L}}{V} = (10.8 \pm 0.7) \left(\frac{gT}{m_D}\right)^2 \alpha^5 T^4 \left[\ln\left(\frac{m_D}{\gamma}\right) + 3.041 + \left(\frac{1}{\ln(1/g)}\right)\right]\ . \tag{18}$$

The overall coefficient has been determined by a numerical lattice simulation [23]. From Eq. (18) one easily obtains the temperature range where sphaleron processes are in thermal equilibrium:

$$T_{EW} \sim 100 \text{ GeV} < T < T_{sph} \sim 10^{12} \text{ GeV}\ . \tag{19}$$

The effective theory describing topological fluctuations of the gauge field in the high-temperature phase is valid for small coupling, $g \ll 1$. Yet for $T_{EW} < T < T_{sph} \sim 10^{12}$ GeV one has $g = \mathcal{O}(1)$. This implies that the electric screening lenth $1/(gT)$ and the magnetic screening length $1/(g^2T)$ are not well separated and that nonperturbative corrections to the sphaleron rate, Eq. (18), may be large. This will modify the temperature range given in Eq. (19), but one expects that the qualitative picture of fluctuations in baryon and lepton number in the high-temperature phase of the Standard Model will not be affected.

2.4 Electroweak Baryogenesis and its Experimental Constraints

An important ingredient in the theory of Baryogenesis is related to the nature of the electroweak transition from the high-temperature symmetric phase to the low-temperature

Higgs phase. Because in the Standard Model baryon number, C and CP are not conserved, it is conceivable that the cosmological baryon asymmetry could have been generated at the electroweak phase transition [13], provided that this transition is of first-order, because then there is also the necessary departure from thermal equilibrium. This possibility has stimulated much theoretical activity during the past years to determine the phase diagram of the electroweak theory.

Electroweak Baryogenesis requires that the baryon asymmetry generated during the phase transition is not erased by sphaleron processes afterwards.[3] This leads to a condition on the jump of the Higgs vacuum expectation value $v_F = \sqrt{H^\dagger H}$ at the critical temperature [24]:

$$\frac{\Delta v_F(T_c)}{T_c} > 1 \,. \tag{20}$$

The strength of the electroweak transition has been studied by numerical and analytical methods as function of the Higgs boson mass. For the $SU(2)$ gauge-Higgs model one finds from lattice simulations as well as perturbative calculations that the lower bound of Eq. (20) is violated for Higgs masses above 45 GeV [25].[4] Because the present lower bound from LEP on the Higgs mass is 114 GeV [26], it is clear that the electroweak transition in the Standard Model is too weak for Baryogenesis. However, for special choices of parameters or by adding singlet fields, in certain circumstances supersymmetric extensions of the Standard Model have a sufficiently strong first-order phase transition to allow Electroweak Baryogenesis to take place [27].

For large Higgs masses, the nature of the electroweak transition is dominated by nonperturbative effects of the $SU(2)$ gauge theory at high temperatures. At a critical Higgs mass $m_H^c = \mathcal{O}(M_W)$, an intriguing phenomenon occurs: The first-order phase transition turns into a smooth crossover [28–30], as expected on general grounds [25]. At the endpoint of a critical line of first-order transitions, which is reached for $m_H = m_H^c$, the phase transition is of second order [31].

The value of the critical Higgs mass can be estimated by comparing the W-boson mass M_W in the Higgs phase with the magnetic mass m_{SM} in the symmetric phase. This yields $m_H^c \simeq 74$ GeV [32]. Numerical lattice simulations have determined the precise value $m_H^c = 72.1 \pm 1.4$ GeV [33]. The analytic estimate of the critical Higgs mass can be generalized to supersymmetric extensions of the Standard Model, where one finds $m_h^c < 130 \ldots 150$ GeV [34], which is still compatible with the present experimental lower bound.

[3] The produced asymmetry will be erased if, after the phase transition, (B+L)-violating processes are in equilibrium.

[4] For Higgs masses below 50 GeV, the Higgs model provides a good approximation for the full Standard Model.

2.5 The Relation Between Baryon and Lepton Asymmetries

In a weakly coupled plasma, one can assign a chemical potential μ to each of the quark, lepton and Higgs fields. In the Standard Model, with one Higgs doublet H and N_f generations one then has $5N_f + 1$ chemical potentials.[5] For a non-interacting gas of massless particles the asymmetry in the particle and antiparticle number densities is given by

$$n_i - \overline{n}_i = \frac{gT^3}{6} \begin{cases} \beta\mu_i + \mathcal{O}\left((\beta\mu_i)^3\right) \ , & \text{fermions} \ , \\ 2\beta\mu_i + \mathcal{O}\left((\beta\mu_i)^3\right) \ , & \text{bosons} \ . \end{cases} \tag{21}$$

The following analysis is based on these relations for $\beta\mu_i \ll 1$. However, one should keep in mind that the plasma of the early Universe is very different from a weakly coupled relativistic gas, owing to the presence of unscreened non-abelian gauge interactions, where nonperturbative effects are important in some cases.

Quarks, leptons and Higgs bosons interact via Yukawa and gauge couplings and, in addition, via the nonperturbative sphaleron processes. In thermal equilibrium all these processes yield constraints between the various chemical potentials [35]. The effective interaction of Eq. (10) induced by the $SU(2)$ electroweak instantons implies

$$\sum_i \left(3\mu_{qi} + \mu_{li}\right) = 0 \ . \tag{22}$$

One also has to take the $SU(3)$ Quantum Chromodynamics (QCD) instanton processes into account [36], which generate an effective interaction between left-handed and right-handed quarks. The corresponding relation between the chemical potentials reads

$$\sum_i \left(2\mu_{qi} - \mu_{ui} - \mu_{di}\right) = 0 \ . \tag{23}$$

A third condition, valid at all temperatures, arises from the requirement that the total hypercharge of the plasma vanishes. From Eq. (21) and the known hypercharges one obtains

$$\sum_i \left(\mu_{qi} + 2\mu_{ui} - \mu_{di} - \mu_{li} - \mu_{ei} + \frac{2}{N_f}\mu_H\right) = 0 \ . \tag{24}$$

The Yukawa interactions, supplemented by gauge interactions, yield relations between the chemical potentials of left-handed and right-handed fermions,

$$\mu_{qi} - \mu_H - \mu_{dj} = 0 \ , \quad \mu_{qi} + \mu_H - \mu_{uj} = 0 \ , \quad \mu_{li} - \mu_H - \mu_{ej} = 0 \ . \tag{25}$$

These relations hold if the corresponding interactions are in thermal equilibrium. In the temperature range 100 GeV $< T <$ 10^{12} GeV, which is of interest for Baryogenesis,

[5] In addition to the Higgs doublet, the two left-handed doublets q_i and ℓ_i and the three right-handed singlets u_i, d_i, and e_i of each generation each have an independent chemical potential.

this is the case for gauge interactions. On the other hand, Yukawa interactions are in equilibrium only in a more restricted temperature range that depends on the strength of the Yukawa couplings. In the following we shall ignore this complication which has only a small effect on our discussion of Leptogenesis.

Using Eq. (21), the baryon number density $n_B \equiv gBT^2/6$ and the lepton number densities $n_{L_i} \equiv L_i g T^2/6$ can be expressed in terms of the chemical potentials:

$$B = \sum_i \left(2\mu_{qi} + \mu_{ui} + \mu_{di}\right) , \tag{26}$$

$$L_i = 2\mu_{li} + \mu_{ei} , \quad L = \sum_i L_i . \tag{27}$$

Consider now the case where all Yukawa interactions are in equilibrium. The asymmetries $L_i - B/N_f$ are then conserved and we have equilibrium between the different generations, $\mu_{li} \equiv \mu_l$, $\mu_{qi} \equiv \mu_q$, etc. Using also the sphaleron relation and the hypercharge constraint, one can express all chemical potentials, and therefore all asymmetries, in terms of a single chemical potential that may be chosen to be μ_l,

$$\begin{aligned} \mu_e &= \frac{2N_f+3}{6N_f+3}\mu_l , \quad \mu_d = -\frac{6N_f+1}{6N_f+3}\mu_l , \quad \mu_u = \frac{2N_f-1}{6N_f+3}\mu_l , \\ \mu_q &= -\frac{1}{3}\mu_l , \quad \mu_H = \frac{4N_f}{6N_f+3}\mu_l . \end{aligned} \tag{28}$$

The corresponding baryon and lepton asymmetries are

$$B = -\frac{4N_f}{3}\mu_l , \quad L = \frac{14N_f^2+9N_f}{6N_f+3}\mu_l . \tag{29}$$

This yields the important connection between the B, $B-L$ and L asymmetries [37]

$$B = c_s(B-L); \quad L = (c_s-1)(B-L) , \tag{30}$$

where $c_s = (8N_f+4)/(22N_f+13)$. The above relations hold for temperatures $T \gg v_F$. In general, the ratio $B/(B-L)$ is a function of v_F/T [38].

The relations (30) between B-, (B-L)- and L-number suggest that (B-L)-violation is needed in order to generate a B-asymmetry.[6] Because the (B-L)-current has no anomaly, the value of B-L at time t_f, where the Leptogenesis process is completed, determines the value of the baryon asymmetry today,

$$B(t_0) = c_s\ (B-L)(t_f) . \tag{31}$$

[6] In the case of Dirac neutrinos, which have extremely small Yukawa couplings, one can construct Leptogenesis models where an asymmetry of lepton doublets is accompanied by an asymmetry of right-handed neutrinos such that the total L-number is conserved and the (B-L)-asymmetry vanishes [39].

On the other hand, during the Leptogenesis process the strength of (B-L)-, and therefore L-violating interactions can only be weak. Otherwise, because of Eq. (30), they would wash out any baryon asymmetry. As we shall see in the following, the interplay between these conflicting conditions leads to important constraints on the properties of neutrinos.

3 Experimental and Theoretical Information on the Neutrino Sector

3.1 Results from Oscillation Experiments

The search for neutrino mass has a long history [40]. Positive results are now provided by neutrino oscillation experiments. The allowed values for the mass-squared differences Δm_{ij}^2 and the mixing angles θ_{ij} at the 3σ level for three generations of neutrinos are summarized below [26]:

$$\sin^2 2\theta_{23} = 0.92 - 1, \quad |\Delta m_{23}^2| = (1.2 - 4.8) \times 10^{-3}\mathrm{eV}^2 \quad (\text{atmospheric } \nu) \tag{32}$$

$$\sin^2 2\theta_{12} = 0.70 - 0.95, \quad |\Delta m_{12}^2| = (5.4 - 9.5) \times 10^{-5}\mathrm{eV}^2 \quad (\text{solar } \nu). \tag{33}$$

The CHOOZ experiment [41] gives only an upper limit on the remaining mixing angle θ_{13}:

$$\sin^2 2\theta_{13} = 0 - 0.17 \quad (\text{CHOOZ } \nu). \tag{34}$$

The LSND experiment [42] reports neutrino oscillations from $\bar{\nu}_\mu$ to $\bar{\nu}_e$. The mixing angle and mass-squared difference inferred from this experiment are $\sin^2 2\theta = 0.003 - 0.03$ and $|\Delta m^2| = 0.2 - 2$ eV^2. This parameter region is almost excluded by negative results from a comparable experiment by the KARMEN collaboration [43], but there still remains a narrow region allowed at the 90% CL.

It is very difficult to explain all the above data from neutrino oscillation experiments within the three neutrino framework. Indeed, to accommodate the LSND data one must introduce at least one sterile neutrino [44], or make the radical assumption that CPT is not conserved [45]. In this review, for simplicity we will disregard the data from the LSND experiment.

3.2 Information from β-decay, 2β-decay and Cosmology

The oscillation experiments discussed in the previous subsection are only sensitive to mass-squared differences. In this subsection we quote results from direct searches for the absolute values of neutrino masses.

The direct laboratory limits on the neutrino masses are summarized as follows [26]:

$$m_{\nu_e} < 2.5 \text{ eV} \; ; \tag{35}$$

$$m_{\nu_\mu} < 170 \text{ keV} \; ; \tag{36}$$

$$m_{\nu_\tau} < 18 \text{ MeV} \; . \tag{37}$$

The ν_e mass measurements use the decay of tritium, $^3\text{H} \to ^3 \text{He} + \bar{\nu}_e + e^-$, which has a small Q value, $Q = 18.6$ keV, and looks at the electron spectrum near the end point in the Kurie plot. The limit on the ν_μ mass is obtained from the two-body kinematics of the pion decay, $\pi^+ \to \mu^+ + \nu_\mu$. Finally, the limit on the ν_τ mass is obtained from measurements of the invariant mass distribution of 3π and 5π systems in the τ decays, $\tau \to 3(5)\pi + \nu_\tau$.

Neutrinoless double β decay experiments provide a bound on an element of the Majorana mass matrix, $m_{\nu_{ee}}$ [46]. The best limit comes from the ^{76}Ge results. Because the calculation of the double β decay rate is model dependent, we quote a range for this bound:

$$m_{\nu_{ee}} < 0.3 - 0.8 \text{ eV} \; . \tag{38}$$

One can derive a stringent upper limit on the sum of neutrino masses from cosmology. In the early Universe neutrinos were in thermal equilibrium with radiation and one can infer their number density today to be $n_i \simeq 110 \text{ cm}^{-3}$ for each neutrino species. Although the contribution of massless neutrinos to the Universe's energy density is negligible, the contribution of massive neutrinos could be important. By requiring that the energy density of massive neutrinos does not exceed that of dark matter, one obtains the bound:

$$\sum_i m_i < 30h^2 \text{ eV}, \tag{39}$$

where h is the Hubble constant in units of 100 km $\text{s}^{-1}\text{Mpc}^{-1}$ [$h = 0.71^{+0.04}_{-0.03}$ [4]].

Even if neutrinos satisfy the above limit, massive neutrinos would affect the formation of cosmic structure, because the free streaming of neutrinos suppresses density fluctuations at small scales. The normalization of large- and small-scale fluctuations constrains the contribution from neutrinos. Recent detailed analyses [47] lead to the bound:

$$\sum_i m_i < 0.65 \text{ eV}. \tag{40}$$

It is interesting that a similar constraint, $\sum_i m_i < 2.0$ eV, has been obtained by using the cosmic microwave background data alone [48].

3.3 The Seesaw Mechanism

If there are right-handed neutrinos, then neutrinos can have a Dirac mass much as quarks and charged leptons do. However, if this is the only source for their mass, it is not easy to find a natural reason for the very small mass of neutrinos. With only a Dirac mass term, the smallness of neutrino masses needs to be ascribed to having very tiny Yukawa coupling constants h ($h \sim 10^{-13}$ gives a neutrino mass $m_\nu \simeq 0.01$ eV). Although it is possible to imagine mechanisms that result in very small Dirac masses for neutrinos,[7] the seesaw mechanism, which entails Majorana masses, provides a natural explanation for the smallness of neutrino masses in theories of unification at ultra-high energy scales [14].

To appreciate this point, we should first note that the Standard Model does not require neutrinos to be massive, because right-handed neutrinos are not necessary for the electroweak theory. Without right-handed neutrinos, these particles may acquire their mass only from what are called irrelevant operators – operators such as $\ell\ell HH$ with dimension greater than four. These operators can give rise to a Majorana mass for neutrinos in theories with a cutoff. However, in the limit of an infinite cutoff, neutrino masses vanish. It should be noted that if one adopts the Planck scale as the Wilson cutoff for the Standard Model, one finds neutrino masses to be at most 10^{-5} eV. Thus, the observed mass $\sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$ eV is unable to be explained within the Standard Model.

If one considers possible extensions of the Standard Model gauge group, it is natural to consider a gauge group G whose rank is at least 5, since the rank of the Standard Model group, $SU(3)\times SU(2)\times U(1)_Y$, is 4. This new group G may contain an extra $U(1)$ as a subgroup, in addition to the Standard Model gauge group. The simplest candidate for the extra $U(1)$ is a B-L gauge symmetry, because we know that the global $U(1)_{B-L}$ does not have any anomaly due to the Standard Model gauge interactions. However, when one gauges this B-L symmetry, the theory does have a self-anomaly of the B-L interactions. That is, the triangle anomaly of $[U(1)_{B-L}]^3$ is non-vanishing. A crucial point, however, is that this anomaly is cancelled by introducing right-handed neutrinos. This also cancels the mixed gravitational/B-L anomaly. Thus, right-handed neutrinos are required for consistency of the theory! A famous example which includes $U(1)_{B-L}$ is provided by $SO(10)$ grand unification. But our argument is more general. For instance, the string brane world predicts many $U(1)$'s and it is quite natural to consider some of them to be anomaly free and survive as low-energy (compared with the string scale) gauge symmetries.

It is usually assumed that the unification group G is broken down to the Standard

[7] At the end of this subsection we outline a recent approach that may explain how such extremely small Yukawa coupling constants might arise in a higher dimensional theory.

Model group at high energies. Then, the right-handed neutrinos naturally obtain large Majorana masses, because they are singlets of the Standard Model and there is no unbroken symmetry to protect them from acquiring a large Majorana mass. In this article we shall denote heavy Majorana neutrinos as N. Then, the masses of the neutrinos written as a matrix take a simple form:

$$\begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} . \tag{41}$$

Here m is the Dirac mass matrix between the left-handed neutrinos ν and the heavy Majorana neutrinos N, which is of order the electroweak scale, while M is the Majorana mass matrix of the heavy neutrinos N. For three generations, both M and m are 3×3 matrices. Integrating out the heavy Majorana neutrinos N leads to a small neutrino mass via the seesaw mechanism [14]. For one neutrino generation one simply has that:

$$m_\nu \simeq \frac{m^2}{M}. \tag{42}$$

We see from the above that a small neutrino mass is a reflection of the ultra-heavy mass of the heavy neutrino N. The observed neutrino mass $\sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$ eV implies $M \simeq 10^{15}$ GeV, which is very close to the Grand Unification scale. Thus, effectively, the small neutrino masses provide a window to new physics at an ultra-high energy scale.

One may question why the unification group G, or the $U(1)_{B-L}$ symmetry, should be broken at such a high energy scale. Perhaps the answer to this question, as we shall amplify in this review, is because, otherwise, the baryon number in the Universe would be too small for us to exist! It turns out that if the Universe's baryon number were to be two orders of magnitude below the present observed value galaxies would not be formed [49]. One of the purposes of this review article is to explain in some detail this fundamental point.

3.3.1 Small Neutrino Yukawa Couplings from Higher Dimensional Theories

To explain how one can generate small Dirac masses for neutrinos, consider a theory described in (4+1)-dimensional space-time, while our world is on a (3+1)-dimensional hyperplane – a so-called D3 brane. The Einstein action of gravity in five-dimensional space-time is given by

$$S = \frac{M_*^3}{16\pi} \int d^4x \int dy \sqrt{-g_5} \mathcal{R}_5, \tag{43}$$

where M_* is the gravitational scale in five-dimensional space-time, and g_5 and $\mathcal{R}_5$ are the metric and the scalar curvature, respectively. We assume that the fifth dimension is compactified to a space of radius L, and consider the metric to be

$$d^2s = g_{\mu\nu} dx^\mu dx^\nu - dy^2, \tag{44}$$

where $g_{\mu\nu}$ is the metric in four-dimensional space-time. The integration over dy leads to the four-dimensional action

$$S_4 = \frac{M_*^3 L}{16\pi} \int d^4x \sqrt{-g_4} \mathcal{R}_4. \tag{45}$$

Then, the Planck scale, $M_{\rm P} \simeq 1.2 \times 10^{19}$ GeV, in four-dimensional space-time is given by

$$M_{\rm P}^2 = M_*^3 L. \tag{46}$$

One can get the observed value for $M_{\rm P}$ even for $M_* = 1$ TeV by taking a very large L. The weakness of gravity in these theories is the result of having a large compactification scale in the fifth dimension [50].

Let us now assume that all the Standard Model particles reside on a D3 brane at the boundary $y = 0$ and that the right-handed neutrino lives in the five-dimensional bulk [51]. Then the action involving the right-handed neutrinos is given by

$$S = M_* \int d^4x \int_0^L dy \sqrt{-g_5} \bar{N}_R i \partial\!\!\!/ N_R + \int d^4x \int_0^L dy \sqrt{-g_5} h \bar{N}_R \ell_L H \delta(y) + \text{h.c.}\,, \tag{47}$$

Here, ℓ_L and H are $\mathrm{SU}(2)_L$ doublets of the left-handed leptons and of the Higgs boson, respectively. One obtains the action in four dimensions by integrating over dy, and one finds:

$$S_4 = M_* L \int d^4x \sqrt{-g_4} \bar{N}_R i \partial\!\!\!/ N_R + \int d^4x \sqrt{-g_4} h \bar{N}_R \ell_L H + \text{h.c.}\,. \tag{48}$$

After renormalizing the wave function of N_R so that it has a canonical kinetic term, one finds that the Yukawa coupling constant is suppressed by $1/\sqrt{M_* L} = M_*/M_{\rm P}$. Thus in this model one obtains an effective Yukawa coupling constant $h_{\rm eff} \simeq 10^{-13}$ (corresponding to a neutrino Dirac mass $m_\nu \simeq 0.01$ eV) for $h = 1$ and $M_* \simeq 10^3$ TeV.

This model, however, has a serious drawback. There is no symmetry to protect the right-handed neutrinos from acquiring a Majorana mass, because they are singlets of the Standard Model gauge group. A solution to this problem may be found by imposing a $U(1)_{B-L}$ gauge symmetry in the five-dimensional bulk. As long as the B-L gauge symmetry is exact, the right-handed neutrinos cannot have a Majorana mass because this mass carries a non-vanishing B-L charge. If this symmetry is exact, the corresponding gauge boson is completely massless. However, this may not cause any phenomenological difficulties at low-energies, because the B-L gauge coupling constant must also be extremely suppressed.[8] But, as we explained in Section 2, if the B-L gauge symmetry is exact, it is very difficult to account for the baryon-number asymmetry in the present Universe.

[8]The B-L gauge coupling constant α_{B-L} is constrained to be $\alpha_{B-L} < 10^{-21}\alpha_{\rm em}$. This bound comes from the empirical limits of the electromagnetic charges for the neutron and the neutrino. That is, $Q_n = (-0.4 \pm 1.1) \times 10^{-21}$ [52] and $Q_\nu = (0.5 \pm 2.9) \times 10^{-21}$ [53]. The present model suggests $\alpha_{B-L} \simeq (\frac{M_*}{M_P})^2 \times \alpha_{\rm em} \simeq 10^{-25} \times \alpha_{\rm em}$, which may be in an interesting region for future experiments.

3.4 CP-Violating Phases at Low and High Energies in the Lepton Sector

In the Standard Model, the Lagrangian for the lepton sector, augmented by including right handed neutrinos, is given by

$$\begin{aligned} \mathcal{L} &= \bar{\ell}_{Li} i\partial\!\!\!/ \ell_{Li} + \bar{e}_{Ri} i\partial\!\!\!/ e_{Ri} + \bar{N}_{Ri} i\partial\!\!\!/ N_{Ri} \\ &\quad + f_{ij}\bar{e}_{Ri}\ell_{Lj}H^\dagger + h_{ij}\bar{N}_{Ri}\ell_{Lj}H - \frac{1}{2}M_{ij}N_{Ri}N_{Rj} + \text{h.c.}\,, \end{aligned} \tag{49}$$

where $i, j = \{1-3\}$ are the family-number indices. We adopt, without losing generality, a basis where the matrices f_{ij} and M_{ij} are diagonal. The Yukawa matrix h_{ij} in this basis is in general complex and thus has CP-violating phases. Because for three families, the matrix h_{ij} has 9 complex parameters, we have 9 possible CP-violating phases. However, three of these phases can be absorbed into the wave function of ℓ_L and hence 6 CP-violating phases remain physically relevant. These are known as high-energy phases, because they enter in the full theory.[9]

Let us now discuss the CP-violating phases at low energies. To do that we first need to integrate out the heavy Majorana neutrinos, N_i. Doing so the effective Lagrangian for the lepton sector reduces to:

$$\mathcal{L}_{\text{eff}} = \bar{\ell}_{Li} i\partial\!\!\!/ \ell_{Li} + \bar{e}_{Ri} i\partial\!\!\!/ e_{Ri} + f_{ii}\bar{e}_{Ri}\ell_{Li}H^\dagger + \frac{1}{2}\sum_k h^T_{ik}h_{kj}\ell_{Li}\ell_{Lj}\frac{H^2}{M_k} + \text{h.c.}\,. \tag{50}$$

The last term can be rewritten as

$$-\frac{1}{2}m_{\nu_{ij}}\ell_{Li}\ell_{Lj}\frac{H^2}{\langle H\rangle^2}\,, \tag{51}$$

so that all low-energy phases appear in the mass matrix of light neutrinos. Because the neutrino mass matrix is symmetric, it has 6 complex parameters, and hence one has 6 possible CP-violating phases. However, as before, 3 of these 6 phases can be absorbed into the wave function of ℓ_L. Therefore, there remain only three physical low energy CP-violating phases [54]. Because they are different in number, it is unfortunately very difficult to establish a direct link between the low-energy and the high-energy CP-violating phases [55].

Furthermore, in practice, it is not possible to measure all three low-energy phases. One of these three phases can be measured by neutrino oscillation experiments, while neutrinoless double β decay, if it were to be observed, would provide information on another phase. However, the remaining phase is undetermined. In other words, one cannot

[9] In particular, as we will show in the next Section, the phase contributing to the generation of the lepton asymmetry in the decay of N_1 is a combination of these high-energy phases, given by $\sum \text{Im}[(hh^\dagger)_{1i}]^2$.

perform a complete experiment to determine the neutrino mass matrix. Nevertheless, if the neutrino mass matrix were to have an extra constraint, one may be able to determine all matrix elements of m_ν. This constraint must be independent of the frame of the family basis. One example of such a constraint is the requirement that $\det(m_\nu) = 0$. In this case, we have only 7 independent physical parameters including the phases in the neutrino mass matrix [56], which can be determined in principle in future experiments.[10]

4 Thermal Leptogenesis

4.1 Lepton Number Violation and Leptogenesis

As we discussed above, lepton number violation is most simply realized by adding right-handed neutrinos to the Standard Model. Their existence is predicted by all extensions of the Standard Model containing B-L as a local symmetry and allows for an elegant explanation of the smallness of the light neutrino masses via the seesaw mechanism [14].

The most general Lagrangian for couplings and masses of charged leptons and neutrinos is given in Eq. (49). The vacuum expectation value of the Higgs field, $\langle H \rangle = v_F$, generates Dirac masses m_e and m_D for charged leptons and neutrinos, $m_e = f v_F$ and $m_D = h v_F$, which are assumed to be much smaller than the Majorana masses M. This yields the light and heavy neutrino mass eigenstates

$$\nu \simeq V_\nu^T \nu_L + \nu_L^c V_\nu^* \quad , \qquad N \simeq N_R + N_R^c \, , \tag{52}$$

with masses

$$m_\nu \simeq -V_\nu^T m_D^T \frac{1}{M} m_D V_\nu \quad , \quad m_N \simeq M \, . \tag{53}$$

In a basis where the charged lepton mass matrix m_e and the Majorana mass matrix M are diagonal, V_ν is the mixing matrix in the leptonic charged current.

The right-handed neutrinos can efficiently erase any pre-existing lepton asymmetry at temperatures $T > M$, but they can also generate a lepton asymmetry by means of their out-of-equilibrium decays at temperatures $T < M$. This asymmetry is then partially transformed into a baryon asymmetry by sphaleron processes. This is the Leptogenesis mechanism proposed by Fukugita and Yanagida [12].

The decay width of the heavy neutrino N_i at tree level reads,

$$\Gamma_{Di} = \Gamma\left(N_i \to H + \ell_L\right) + \Gamma\left(N_i \to H^\dagger + \ell_L^\dagger\right) = \frac{1}{8\pi}(hh^\dagger)_{ii} M_i \, . \tag{54}$$

[10]As will be shown in Section 6, Affleck-Dine Leptogenesis suggests a constraint, $m_{\nu_1} \simeq 10^{-9}$ eV and hence $\det(m_\nu) \simeq 0$.

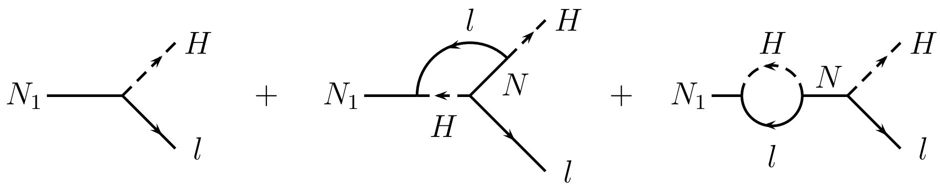

Figure 1: Tree level and one-loop diagrams contributing to heavy neutrino decays whose interference leads to Leptogenesis.

Once the temperature of the universe drops below the mass M_1, the heavy neutrinos are not able to follow the rapid change of the equilibrium distribution. Hence, the necessary deviation from thermal equilibrium ensues as a result of having a too large number density of heavy neutrinos, compared to the equilibrium density. Eventually, however, the heavy neutrinos decay, and a lepton asymmetry is generated owing to the presence of CP-violating processes. The CP asymmetry involves the interference between the tree-level amplitude and the one-loop vertex and self-energy contributions (see Fig. (1)). In a basis, where the right-handed neutrino mass matrix M is diagonal, one obtains [57] for the CP asymmetry parameter ε_1 assuming hierarchical heavy neutrino masses ($M_1 \ll M_2, M_3$):

$$\varepsilon_1 \simeq \frac{3}{16\pi} \frac{1}{(hh^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im}\left[\left(hh^\dagger\right)^2_{i1}\right] \frac{M_1}{M_i} \; . \tag{55}$$

In the case of mass differences of order the decay widths, one obtains a significant enhancement from the self-energy contribution [58], although the influence of the thermal bath on this effect is presently unclear.

The CP asymmetry of Eq. (55) can be obtained in a very simple way by first integrating out the heavier neutrinos N_2 and N_3 in the leptonic Lagrangian. This yields

$$\mathcal{L}_\nu^{eff} = h_{1j}\overline{N_{R1}}\ell_{Lj}H - \frac{1}{2}M_1\overline{N^c_{R1}}N_{R1} + \frac{1}{2}\eta_{ij}\ell_{Li}H\ell_{Lj}H + \text{ h.c.} \; , \tag{56}$$

with

$$\eta_{ij} = \sum_{k=2}^{3} h^T_{ik}\frac{1}{M_k}h_{kj} \; . \tag{57}$$

The asymmetry ε_1 is then obtained from the interference of the Born graph and the one-loop graph involving the cubic and the quartic couplings. This includes automatically both, vertex and self-energy corrections [59] and yields an expression for ε_1 directly in terms of the light neutrino mass matrix:

$$\varepsilon_1 \simeq -\frac{3}{16\pi}\frac{M_1}{(hh^\dagger)_{11}v_F^2}\mathrm{Im}\left(h^* m_\nu h^\dagger\right)_{11} \; . \tag{58}$$

The CP asymmetry then leads to a (B-L)-asymmetry [12],

$$Y_{B-L} \simeq -Y_L = -\frac{n_L - n_{\overline{L}}}{s} = -\kappa\frac{\varepsilon_1}{g_*} \; . \tag{59}$$

Here s is the entropy and, in the present epoch $s = 7.04 n_\gamma$, whereas $g_* \sim 100$ is the number of degrees of freedom in the plasma. The factor $\kappa < 1$ in the above takes into account the effect of washout processes. As we shall discuss below, in order to determine κ one has to solve the Boltzmann equations.

Early studies of Leptogenesis were partly motivated by trying to find alternatives to Electroweak Baryogenesis, which did not seem to produce a big enough asymmetry. Some extensions of the Standard Model were considered and, in particular, in the simple case of hierarchical heavy neutrino masses the observed value of the baryon asymmetry is naturally obtained with B-L broken at the unification scale, $M_{GUT} \sim 10^{15}$ GeV. The corresponding light neutrino masses are then very small, $m_{1,2} < m_3 \sim 0.1$ eV, and the typical parameters for the necessary CP asymmetry and the Baryogenesis temperature are $\varepsilon_1 \simeq 10^{-6}$ and $T_B \sim M_1 \sim 10^{10}$ GeV, respectively [60,61].[11] Subsequently, researchers realized that such small neutrino masses are consistent with the small mass differences inferred from the solar and atmospheric neutrino oscillations. This fact has given rise to a strong interest in Leptogenesis in recent years, and a large number of interesting models have been suggested [63].

4.2 Departure from Thermal Equilibrium

Leptogenesis takes place at temperatures $T \sim M_1$. For a decay width small compared to the Hubble parameter, $\Gamma_1(T) < H(T)$, heavy neutrinos are out of thermal equilibrium, otherwise they are in thermal equilibrium [64]. The borderline between the two regimes is given by $\Gamma_1 = H|_{T=M_1}$, which is equivalent to the condition that the effective neutrino mass

$$\widetilde{m}_1 = \frac{(m_D m_D^\dagger)_{11}}{M_1} \tag{60}$$

is equal to the 'equilibrium neutrino mass'

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v_F^2}{M_{\rm P}} \simeq 10^{-3} \text{ eV} . \tag{61}$$

Here we have used the Hubble parameter $H(T) \simeq 1.66\, g_* T^2/M_{\rm P}$ where $g_* = g_{SM} = 106.75$ is the total number of degrees of freedom and $M_{\rm P} = 1.22 \times 10^{19}$ GeV is the Planck mass.

It is quite remarkable that the equilibrium neutrino mass m_* is close to the neutrino masses suggested by neutrino oscillations, $\sqrt{\Delta m^2_{\rm sol}} \simeq 8 \times 10^{-3}$ eV and $\sqrt{\Delta m^2_{\rm atm}} \simeq 5 \times 10^{-2}$ eV. This encourages one to think that it may be possible to understand the

[11] For early work based on $SO(10)$, see [62].

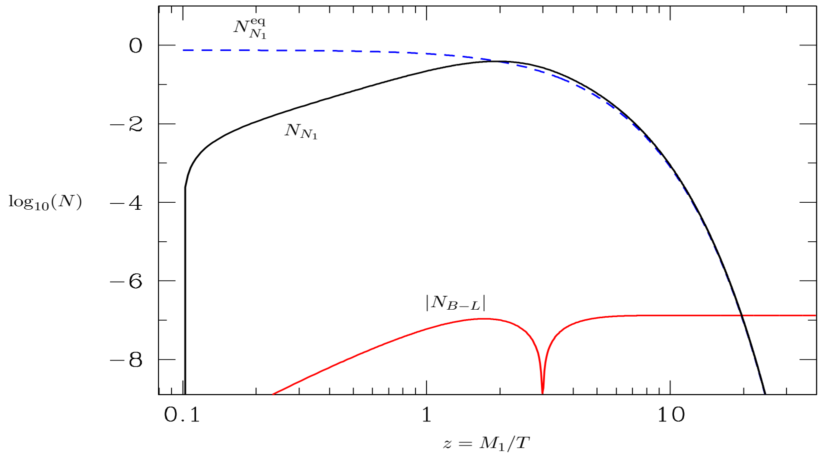

Figure 2: The evolution of the N_1 abundance and the $B - L$ asymmetry for a typical choice of parameters, $M_1 = 10^{10}\,\mathrm{GeV}$, $\varepsilon_1 = 10^{-6}$, $\widetilde{m}_1 = 10^{-3}\,\mathrm{eV}$ and $\overline{m} = 0.05\,\mathrm{eV}$. From [67].

cosmological baryon asymmetry via Leptogenesis as a process close to thermal equilibrium. Ideally, $\Delta L = 1$ and $\Delta L = 2$ processes should be strong enough at temperatures above M_1 to keep the heavy neutrinos in thermal equilibrium and weak enough to allow the generation of an asymmetry at temperatures below M_1.

In general, the generated baryon asymmetry is the result of a competition between production processes and washout processes that tend to erase any generated asymmetry. Unless the heavy Majorana neutrinos are partially degenerate, $M_{2,3} - M_1 \ll M_1$, the dominant processes are decays and inverse decays of N_1 and the usual off-shell $\Delta L = 1$ and $\Delta L = 2$ scatterings [65, 66].

The Boltzmann equations for Leptogenesis are[12]

$$\frac{dN_{N_1}}{dz} = -(D + S)\,(N_{N_1} - N_{N_1}^{\rm eq})\,, \tag{62}$$

$$\frac{dN_{B-L}}{dz} = -\varepsilon_1\, D\,(N_{N_1} - N_{N_1}^{\rm eq}) - W\, N_{B-L}\,, \tag{63}$$

where $z = M_1/T$. The number density N_{N_1} and the amount of $B - L$ asymmetry, N_{B-L}, are calculated in a portion of comoving volume that contains one photon at the onset of Leptogenesis, so that the relativistic equilibrium N_1 number density is given by $N_{N_1}^{\rm eq}(z \ll 1) = 3/4$. Alternatively, one may normalize the number density to the entropy density s and consider $Y_X = n_X/s$. If entropy is conserved, both normalizations are related by a constant.

[12]We use the conventions of [67]. We have also summed over the three lepton flavours neglecting the dependence on the lepton Yukawa couplings [68].

There are four classes of processes that contribute to the different terms in the above equations: decays, inverse decays, $\Delta L = 1$ scatterings and $\Delta L = 2$ processes mediated by heavy neutrinos. The first three processes all modify the N_1 abundance and try to push it towards its equilibrium value $N_{N_1}^{\rm eq}$. Denoting by H the Hubble expansion rate, the term $D = \Gamma_D/(H\,z)$ accounts for decays and inverse decays, whereas the scattering term $S = \Gamma_S/(H\,z)$ represents the $\Delta L = 1$ scatterings. Decays also yield the source term for the generation of the $B-L$ asymmetry, the first term in Eq. (63), whereas all other processes contribute to the total washout term $W = \Gamma_W/(H\,z)$ which competes with the decay source term. The dynamical generation of the N_1 abundance and the $B-L$ asymmetry is shown in Fig. (2) for typical parameters.

4.3 Decays and Inverse Decays

It is very instructive to consider first a simplified picture in which decays and inverse decays are the only processes that are effective.[13] For consistency, in this approximation the real intermediate state contribution to the $2 \to 2$ processes has to be included. In the kinetic equations (62) and (63) one then has to replace $D+S$ by D and W by W_{ID}, respectively, where W_{ID} is the contribution of inverse decays to the washout term. The solution for N_{B-L} in this case is the sum of two terms [64],

$$N_{B-L}(z) = N_{B-L}^{\rm i}\, e^{-\int_{z_{\rm i}}^{z} dz'\, W_{ID}(z')} - \frac{3}{4}\,\varepsilon_1\,\kappa(z;\widetilde{m}_1)\;. \tag{64}$$

Here the first term accounts for an initial asymmetry which is partly reduced by washout, and the second term describes $B-L$ production from N_1 decays. It is expressed in terms of the *efficiency factor* κ [68] which does not depend on the CP asymmetry ε_1,

$$\kappa(z) = \frac{4}{3}\int_{z_{\rm i}}^{z} dz'\, D\left(N_{N_1} - N_{N_1}^{\rm eq}\right)\, e^{-\int_{z'}^{z} dz''\, W_{ID}(z'')}\;. \tag{65}$$

As we shall see, decays and inverse decays are sufficient to describe qualitatively many properties of the full problem.

We will first study in detail the regimes of weak and strong washout. If just decays and inverse decays are taken into account, these regimes correspond, respectively, to the limits $K \ll 1$ and $K \gg 1$ of the decay parameter

$$K = \frac{\Gamma_D(z=\infty)}{H(z=1)} = \frac{\widetilde{m}_1}{m_*}\,, \tag{66}$$

introduced in the context of ordinary GUT baryogenesis [64]. Based on the insight into the dynamics of the non-equilibrium process gained from these limiting cases one can

[13] This section follows closely [69].

then obtain analytic interpolation formulas that describe rather accurately the entire parameter range.

To proceed, let us first recall some basic definitions and formulas. The decay rate is given by the formula [70],

$$\Gamma_D(z) = \Gamma_{D1} \left\langle \frac{1}{\gamma} \right\rangle , \tag{67}$$

where the thermally averaged dilation factor is given by the ratio of the modified Bessel functions K_1 and K_2,

$$\left\langle \frac{1}{\gamma} \right\rangle = \frac{K_1(z)}{K_2(z)} . \tag{68}$$

For the decay term D, one then obtains

$$D(z) = K\, z \left\langle \frac{1}{\gamma} \right\rangle . \tag{69}$$

The inverse decay rate is related to the decay rate by

$$\Gamma_{ID}(z) = \Gamma_D(z)\, \frac{N_{N_1}^{\rm eq}(z)}{N_l^{\rm eq}} , \tag{70}$$

where $N_l^{\rm eq}$ is the equilibrium density of lepton doublets. Because the number of degrees of freedom for heavy Majorana neutrinos and lepton doublets is the same, $g_{N_1} = g_l = 2$, one has

$$N_{N_1}^{\rm eq}(z) = \frac{3}{8} z^2 K_2(z) , \quad N_l^{\rm eq} = \frac{3}{4} . \tag{71}$$

This yields for the contribution of inverse decays to the washout term W:

$$W_{ID}(z) = \frac{1}{2} D(z)\, \frac{N_{N_1}^{\rm eq}(z)}{N_l^{\rm eq}} . \tag{72}$$

All relevant quantities are given in terms of the Bessel functions K_1 and K_2, which can be approximated by simple analytical expressions.

In the regime *far out of equilibrium*, $K \ll 1$, decays occur at very small temperatures, $z \gg 1$, and the produced $(B-L)$-asymmetry is not reduced by washout effects. In this case, using Eq. (62) with $S = 0$, the integral for the efficiency factor given in Eq. (65) becomes simply,

$$\kappa(z) \simeq \frac{4}{3} \left(N_{N_1}^{\rm i} - N_{N_1}(z) \right) . \tag{73}$$

The final value of the efficiency factor $\kappa_{\rm f} = \kappa(\infty)$ is proportional to the initial N_1 abundance. If $N_1^{\rm i} = N_1^{\rm eq} = 3/4$, then $\kappa_{\rm f} = 1$. But if the initial abundance is zero, then $\kappa_{\rm f} = 0$ as well. Therefore, in this region there is the well known problem that one has to invoke some external mechanism to produce the initial abundance of neutrinos. Moreover, an initial (B-L)-asymmetry is not washed out. Thus in the regime $K \ll 1$ the results strongly depend on the initial conditions and there is little predictivity.

In order to obtain the efficiency factor in the case of *vanishing initial N_1-abundance*, $N_{N_1}(z_\mathrm{i}) \equiv N^\mathrm{i}_{N_1} \simeq 0$, one has to calculate how heavy neutrinos are dynamically produced by inverse decays. This requires solving the kinetic equation Eq. (62) with the initial condition $N^\mathrm{i}_{N_1} = 0$.

Let us define a value z_eq by the condition

$$N_{N_1}(z_\mathrm{eq}) = N^\mathrm{eq}_{N_1}(z_\mathrm{eq}) \; . \tag{74}$$

Then Eq. (62) implies that the number density reaches its maximum at $z = z_\mathrm{eq}$. For $z > z_\mathrm{eq}$ the efficiency factor is always the sum of two contributions,

$$\kappa_\mathrm{f}(z) = \kappa^-(z) + \kappa^+(z) \; . \tag{75}$$

Here $\kappa^-(z)$ and $\kappa^+(z)$ correspond to the integration domains $[z_\mathrm{i}, z_\mathrm{eq})$ and $[z_\mathrm{eq}, z)$, respectively.

Consider first the case of *weak washout*, $K \ll 1$, which implies $z_\mathrm{eq} \gg 1$. One then finds,

$$N_{N_1}(z_\mathrm{eq}) \simeq \frac{9\pi}{16}\, K \; . \tag{76}$$

It turns out that to first order in K, there is a cancellation between κ^+ and κ^-, yielding for the final efficiency factor

$$\kappa_\mathrm{f}(K) \simeq \frac{9\pi^2}{64}\, K^2 \; . \tag{77}$$

Note, that Eq. (77) does not hold for $K > 1$, because in this case z_eq becomes small, and washout effects change the result.

In the case of *strong washout*, $K \gg 1$, we can neglect the negative contribution κ^-, because the asymmetry generated at high temperatures is efficiently washed out. Now the neutrino abundance tracks closely the equilibrium behavior. Because $D \propto K$, one can solve Eq. (62) systematically in powers of $1/K$, which yields

$$D\left(N_{N_1}(z) - N^\mathrm{eq}_{N_1}(z)\right) = \frac{3}{2Kz} W_{ID}(z) + \mathcal{O}\left(\frac{1}{K}\right) \; , \tag{78}$$

where we have used properties of the Bessel functions. From Eqs. (65) and (78) one obtains for the efficiency factor[14]

$$\kappa(z) = \frac{2}{K}\int_{z_\mathrm{eq}}^{z} dz' \, \frac{1}{z'} W_{ID}(z')\, e^{-\int_{z'}^{z} dz''\, W_{ID}(z'')} \; . \tag{79}$$

[14]Because κ^- does not contribute we can take the lower limit below as z_i.

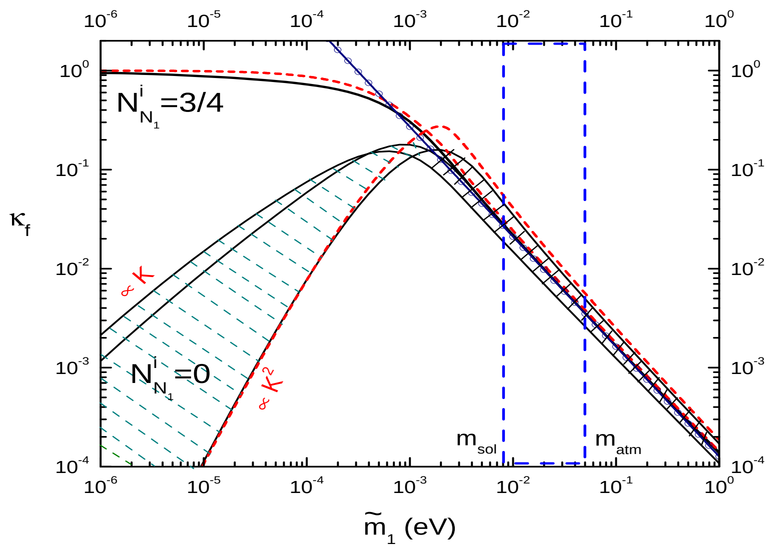

Figure 3: Final efficiency factor when the washout term ΔW is neglected. From [69].

The integral is dominated by the contribution from a region around the value $z_{\rm B}$ where the integrand has a maximum, which is determined by the condition

$$W_{ID}(z_{\rm B}) = \left\langle\frac{1}{\gamma}\right\rangle^{-1}(z_{\rm B}) - \frac{3}{z_{\rm B}}\,. \tag{80}$$

For $K \gg 1$ one has $z_{\rm B} \gg 1$, and the condition (80) becomes approximately $W_{ID}(z_{\rm B}) \simeq 1$, with $W_{ID}(z) > 1$ for $z < z_{\rm B}$ and $W_{ID}(z) < 1$ for $z > z_{\rm B}$. This means that the asymmetry produced for $z < z_{\rm B}$ is essentially erased, whereas for $z > z_{\rm B}$, washout is negligible. Hence, the expression of Eq. (79) is a good approximation for the final efficiency factor.

One finds that a rather accurate expression for $z_B(K)$ is given by

$$z_B(K) \simeq 1 + \frac{1}{2}\ln\left(1 + \frac{\pi K^2}{1024}\left[\ln\left(\frac{3125\pi K^2}{1024}\right)\right]^5\right)\,. \tag{81}$$

The integral of Eq. (79), which gives the final efficiency factor in terms of $z_B(K)$, is well approximated by

$$\kappa_{\rm f}(K) \simeq \frac{2}{z_{\rm B}(K)K}\left(1 - e^{-\frac{1}{2}z_{\rm B}(K)K}\right)\,. \tag{82}$$

Both equations can also be extrapolated into the regime of weak washout, $K \ll 1$, where one obtains $\kappa_{\rm f} = 1$ corresponding to thermal initial abundance, $N^{\rm i}_{N_1} = N^{\rm eq}_{N_1} = 3/4$. At $K \simeq 3$ a rapid transition takes place from strong to weak washout. Even here analytical and numerical results agree within 30%. For the case of zero initial N_1 abundance one obtains an interpolation formula $\kappa_{\rm f}(K)$ analogous to Eq. (82).

The above discussion of decays and inverse decays can be extended to include $\Delta L = 1$ and $\Delta L = 2$ scattering and washout processes. In the weak washout regime, $K \ll 1$, the main effect is that the efficiency factor of Eq. (77) is enhanced to $\kappa_{\rm f} \propto K$. Relevant effects include scattering processes involving gauge bosons [71, 72] and thermal corrections to the decay and scattering rates [72, 73]. The range of different results is represented in Fig. (3) by the hatched region. An additional uncertainty in the weak washout regime is due to the dependence of the final results on the initial N_1 abundance and a possible initial asymmetry created before the onset of Leptogenesis.

The situation is very different in the strong washout regime. Here the final efficiency factor is not sensitive to the neutrino production because a thermal neutrino distribution is always reached at high temperatures. For $\widetilde{m}_1 > m_* \simeq 10^{-3}\,\text{eV}$, the effect of $\Delta L = 1$ processes on the washout is not larger than about 50%, as indicated by the hatched region in Fig. (3). Within these uncertainties, the final efficiency factor is given by the simple power law:

$$\kappa_{\rm f} = (2 \pm 1)\, 10^{-2} \left(\frac{0.01\,\text{eV}}{\widetilde{m}_1}\right)^{1.1\pm 0.1} . \tag{83}$$

Both the scale of solar neutrino oscillations, $m_{\rm sol} \equiv \sqrt{\Delta m^2_{\rm sol}} \simeq 8 \times 10^{-3}\,\text{eV}$, and the scale of atmospheric neutrino oscillations, $m_{\rm atm} \equiv \sqrt{\Delta m^2_{\rm atm}} \simeq 0.05\,\text{eV}$, are larger than the equilibrium neutrino mass m_*. Hence, the range of neutrino masses, and therefore $\widetilde{m}_1$, indicated by neutrino oscillations lies entirely in the strong washout regime where theoretical uncertainties are small and the efficiency factor is still large enough to allow for successful Leptogenesis.

4.4 Bounds on Neutrino Masses

The $\Delta L = 2$ processes with heavy neutrino exchange generate a contribution to the washout rate that depends on the absolute neutrino mass scale $\overline{m}^2 = m_1^2 + m_2^2 + m_3^2$,

$$\Delta W \propto \frac{M_{\rm P} M_1\, \overline{m}^2}{v_F^4} . \tag{84}$$

As long as ΔW can be neglected, the efficiency factor is independent of M_1. With increasing $\overline{m}$ however, the washout rate ΔW becomes important and eventually prevents successful Leptogenesis. This leads to the upper bound on the absolute neutrino mass scale. [67, 74]

One can also obtain a lower bound on the heavy neutrino masses [75], because the CP asymmetry ε_1 satisfies an upper bound [75–78], which is a function of M_1, $\widetilde{m}_1$ and $\overline{m}$. Since the rates entering the Boltzmann equations depend on the same quantities, there exists for arbitrary neutrino mass matrices a maximal baryon asymmetry $\eta_B^{\max}$,

$$\begin{aligned}\eta_B &\le \eta_B^{\max}(\widetilde{m}_1, M_1, \overline{m}) \\ &\simeq 0.01\ \varepsilon_1^{\max}(\widetilde{m}_1, M_1, \overline{m})\ \kappa(\widetilde{m}_1, M_1\overline{m}^2)\ .\end{aligned} \tag{85}$$

Requiring the maximal baryon asymmetry to be larger than the observed one,

$$\eta_B^{\max}(\widetilde{m}_1, M_1, \overline{m}) \ge \eta_B^{CMB}\ , \tag{86}$$

then yields a constraint on the neutrino mass parameters $\widetilde{m}_1$, M_1 and $\overline{m}$. For each value of $\overline{m}$ one obtains a domain in the ($\widetilde{m}_1$-M_1)-plane, which is allowed by successful baryogenesis. For $\overline{m} \ge 0.20$ eV this domain shrinks to zero. One can easily translate this bound into upper limits on the individual neutrino masses. In a similar way, one finds a lower bound on M_1, the smallest mass of the heavy Majorana neutrinos. The resulting upper and lower bounds are [77]

$$m_i < 0.1\,\mathrm{eV}\ , \quad M_1 > 4 \times 10^8\ \mathrm{GeV}\ , \tag{87}$$

where we have assumed thermal initial N_1 abundance. The upper bound on the light neutrino masses holds for a normal as well as an inverted hierarchy of masses. For zero initial N_1 abundance one obtains the more restrictive lower bound $M_1 > 2 \times 10^9$ GeV. For $\widetilde{m}_1 > m_*$, the baryon asymmetry is generated at the temperature $T_B \simeq M_1/z_B < M_1$. Hence the lower bound on the reheating temperature T_i is less restrictive than the lower bound on M_1. The results of a detailed analytical and numerical calculation are summarized in Fig. (4). For the lower bound on the reheating temperature one finds $T_i > 2 \times 10^9$ GeV [69, 72].[15]

What is the theoretical error on the upper bound for the light neutrino masses? In order to answer this question one needs a full quantum mechanical treatment of Leptogenesis, a challenging problem! A possible starting point is the Kadanoff-Baym equations for which a systematic expansion around the Boltzmann equations can be constructed [59]. One then has to calculate relativistic corrections, off-shell effects, 'memory effects', higher order loop corrections, etc. One important effect is the running of neutrino masses between the Fermi scale and the energy scale of Leptogenesis [68, 81]. Also relevant are the $\Delta L = 1$ scattering processes involving gauge bosons [71, 72]. Conceptually interesting are thermal corrections at large temperatures, $T > M_1$, which correspond to loop corrections involving gauge bosons and the top quark [72]. Their effect is large if thermal masses are treated as kinematical masses in the evaluation of scattering matrix elements. At

[15] In the supersymmetric case the CP asymmetry is enhanced but also the washout processes are stronger. These two effects partly compensate each other [79], leading to the slightly less stringent bound $T_i > 1.5 \times 10^9$ GeV [80].

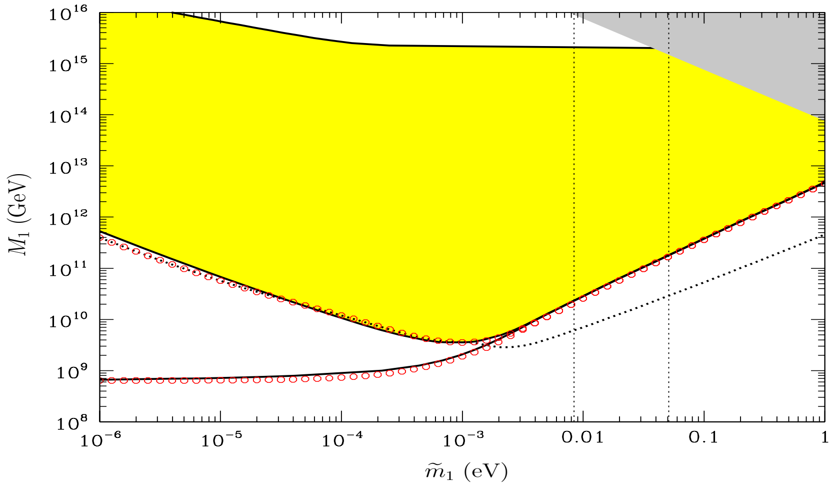

Figure 4: Analytical lower bounds on M_1 (circles) and $T_{\rm i}$ (dotted line) for $m_1 = 0$, $\eta_B^{CMB} = 6 \times 10^{-10}$ and $m_{\rm atm} = 0.05\,{\rm eV}$. The analytical results for M_1 are compared with the numerical ones (solid lines). Upper and lower curves correspond to zero and thermal initial N_1 abundance, respectively. The vertical dashed lines indicate the range $(m_{\rm sol}, m_{\rm atm})$. The gray triangle at large M_1 and large $\widetilde{m}_1$ is excluded by theoretical consistency. From [69].

sufficiently high temperatures the process $N_1 \rightarrow H\ell_L$ is then kinematically forbidden whereas the process $H \rightarrow N_1\ell_L^\dagger$ is allowed by 'phase space'. On the contrary, thermal correction are small if they are only included as propagator effects [73]. It is important to clarify this issue for the treatment of non-equilibrium processes at high temperatures.

The analysis [72] leads to the upper bound on the light neutrino masses $m_i < 0.15$ eV. In [69] an upper bound of 0.12 eV has been obtained. About 0.02 eV of this difference is due to the different treatment of radiative corrections [81], the remaining 0.01 eV reflects differences in the treatment of thermal corrections. This discrepancy has to be compared with an uncertainty of about -0.02 eV due to 'spectator processes' [82], which have not been taken into account in both analyses. Hence, within the minimal seesaw model and the present status of theoretical calculations, the upper bound on the light Majorana neutrino masses is now known rather precisely.

The main result of this section is summarized in Fig. (3). For $\widetilde{m}_1 > m_*$, the efficiency factor, and therefore the baryon asymmetry η_B, is independent of the initial N_1 abundance. Furthermore, the final baryon asymmetry does not depend on the value of an initial baryon asymmetry generated by some other mechanism [77]. Hence, the value of η_B is entirely determined by neutrino properties. In this way Leptogenesis singles out the neutrino mass range

$$10^{-3}\ {\rm eV} < m_i < 0.1\ {\rm eV}\ . \tag{88}$$

The firm predictions of thermal Leptogenesis open a window into the physics of the early universe at temperatures $T_B = \mathcal{O}(10^{10}\ \text{GeV})$, and we can ask what the implications are for dark matter, cosmology and particle physics.

4.5 Triplet Models and Resonant Leptogenesis

Measurements in neutrino physics determine the parameters of the neutrino mass matrix,

$$m_\nu = -m_D^T \frac{1}{M} m_D + m_\nu^{\text{triplet}} , \tag{89}$$

which in general contains a contribution from $SU(2)$ triplet fields [83] in addition to the seesaw term generated by $SU(2)$ singlet heavy Majorana neutrinos. So far, we have only considered the minimal case, $m_\nu^{\text{triplet}} = 0$. Clearly, a dominant triplet contribution would destroy the connection between Leptogenesis and low energy neutrino physics.

The discovery of quasi-degenerate neutrinos with masses above the bound 0.1 eV would require significant modifications of minimal Leptogenesis and/or the seesaw mechanism. In this case $SU(2)$ triplet contributions to neutrino masses could be a possible way out [78, 84, 85]. Clearly, one then has no upper bound on the light neutrino masses anymore. Yet Leptogenesis with right-handed neutrino decays can still work yielding a slightly relaxed lower bound on the heavy neutrino masses. For instance, one may have $m_i \sim 0.35$ eV with $M_1 > 4 \times 10^8$ GeV [85].

Another way to reconcile quasi-degenerate light neutrinos with Leptogenesis makes use of the enhancement of the CP asymmetry in case of quasi-degenerate heavy neutrinos [86]. For instance, to raise the upper bound from 0.1 eV to 0.4 eV, a degeneracy of $\Delta M/M$ for the heavy neutrinos in the range $0.4 - 10^{-3}$ is required, depending on assumptions about the neutrino mass matrices [77, 78]. In the extreme case of 'resonant Leptogenesis' [71], CP asymmetries $\varepsilon = \mathcal{O}(1)$ are reached for degeneracies $\Delta M/M = \mathcal{O}(10^{-10})$. In this case the right-handed neutrino masses may be as small as 1 TeV, which may lead to observable signatures at colliders. A number of models of this type have been constructed [87], some of which make use of the relative smallness of soft supersymmetry breaking terms [88].

4.6 Is the CP Violation in Leptogenesis Connected with the Low Energy CP Violation in the Neutrino Sector?

As was shown in Section 3, the seesaw model has 6 CP-violating phases in the Yukawa matrix h_{ij}. Leptogenesis depends on one combination of these 6 phases. However, there are only 3 CP-violating phases at low energies. Hence it is impossible to determine all

6 phases in the theory, even if one were to measure all 3 low-energy phases. Futhermore, as we discussed earlier, one of these low-energy phases remains undetermined by experiments feasible at low energies.

Nevertheless, the effective number of high energy CP-violating phases is reduced if one of the superheavy Majorana neutrinos N_i is extremely heavy and decouples from the seesaw system. In this case, the Yukawa couplings h_{ij} effectively are given by a 2×3 matrix that contains 6 complex parameters and hence 6 phases. Three of the 6 phases can be absorbed into the wave functions of ℓ_L and thus one is left with only 3 CP-violating phases at high energies. In this case, the 3 low-energy CP-violating phases that appear in the neutrino mass matrix m_ν are reduced to only two physical phases, because $\det(m_\nu) \simeq 0$. Although these 2 low-energy phases are, in principle, measurable in future experiments, this is still not enough to determine all three phases in the full theory. Therefore, even in this simplified example, one cannot establish any link between the sign of the Universe's baryon-number asymmetry with the observable CP-violating phases at low energies.

In the very special case where h_{ij} has two zeros, one has only one CP-violating phase. In this case the CP-violating phase in neutrino oscillations is connected with the phase in Leptogenesis or, equivalently, the sign of the baryon-number asymmetry in the Universe [89].[16] Thus, in this case, one may indeed test directly the idea of Leptogenesis. It is interesting that such a restricted model, where $h_{13} = h_{21} = 0$, is still consistent with data on neutrino oscillations.

5 Dark Matter Considerations

It is certainly possible that the mechanism that generates a primordial matter-antimatter asymmetry in the Universe is not physically related to the existence of a non-luminous component of the energy density of the Universe, a component that now accounts for about 25 % of the total energy density. However, it would be very interesting if these two phenomena, so central to the history of the Universe, were connected in some deep way. It turns out, as we shall see, that if Leptogenesis is the mechanism by which a primordial matter-antimatter asymmetry in the Universe is established, it considerably impacts what the dark matter in the Universe can be.

Of the three viable options for dark matter, from the point of view of particle physics,

[16]The prediction of the sign of the CP-violating phase in neutrino oscillations depends on which heavy Majorana neutrino is responsible for Leptogenesis. This problem is solved in the inflaton-decay scenario in supersymmetry (SUSY) theories, because one choice is unable to produce enough lepton-number asymmetry due to the constraint on the reheating temperature $T_R < 10^7$ GeV [90].

two are either linked or constrained by thermal Leptogenesis and the third has clear connections to nonthermal Leptogenesis. Before discussing these points in some detail, it is useful to briefly review the extant dark matter candidates motivated by particle physics.

5.1 PQ Symmetry and Axions

It is well known that QCD admits the presence of an additional CP violating term in its Lagrangian density [17],

$$\mathcal{L}_\theta = \frac{\alpha_3}{8\pi}\theta F_i^{\mu\nu}\tilde{F}_{i\mu\nu}, \tag{90}$$

where $\tilde{F}_{i\mu\nu} = 1/2\epsilon_{\alpha\beta\mu\nu}F_i^{\alpha\beta}$. If θ is non-vanishing $\mathcal{L}_\theta$, which is C-even and P-odd, violates CP and T invariance. Because possible CP violating parameters of the strong interactions, like the electric dipole moment of the neutron, are very tightly bounded by experiment [91], the parameter θ must be very small ($\theta \leq 10^{-10}$) [92]. The reason for this is a mystery, and is known as the Strong CP Problem [93].

Probably the most 'natural' solution suggested for the Strong CP Problem is to assume that the total Lagrangian for the strong and electroweak interactions is invariant under a global chiral $U(1)_{PQ}$ symmetry [94]. Even though this symmetry is spontaneously broken, one can show [94] that as a result of the $U(1)_{PQ}$ symmetry the parameter θ is driven to zero. In effect, what happens is that the CP violating Lagrangian term $\mathcal{L}_\theta$ is replaced by a CP conserving interaction between the CP odd pseudo-Goldstone boson[17] associated with the spontaneous breakdown of $U(1)_{PQ}$ – the axion – [95] and $F\tilde{F}$:

$$\mathcal{L}_\theta \to \frac{\alpha_3}{8\pi}\frac{a}{f_a}F_i^{\mu\nu}\tilde{F}_{i\mu\nu}. \tag{91}$$

Originally, it was supposed [94] that the $U(1)_{PQ}$ symmetry was broken at the electroweak scale. Then $f_a \sim v_F \simeq 200$ GeV and the axion mass lies in the keV range. However, axions in this mass range, which are coupled with strength $1/f_a$, have been ruled out by experiment [93]. Astrophysical considerations, however, impose very strong constraints on axions much lighter than a keV, as their emission from stars would significantly alter their properties. Only axions that are sufficiently weakly coupled (hence, with large enough f_a and thus a correspondingly small mass) avoid these constraints, and one finds the bound [96] $f_a \geq 10^{10}$ GeV. On the other hand, f_a cannot be arbitrarily large, because zero-momentum axion oscillations in the early Universe would carry

[17] Axions are not true Goldstone bosons because the $U(1)_{PQ}$ symmetry is anomalous [95]. In fact, the same effective potential for axions that serves to drive θ to zero gives axions a small mass. This mass is of order [93] $m_a^2 \sim m_q\Lambda_{QCD}^3/f_a^2$, where f_a is the scale where the $U(1)_{PQ}$ symmetry breaks down spontaneously, and m_q is the (light) quark mass.

enough energy density (proportional, approximately, to f_a) to overclose the Universe [97]. Thus, for an appropriate value for f_a, axions can be the dark matter in the Universe. In particular, one finds [98] that $f_a \simeq 10^{12}$ GeV gives $\Omega_a \simeq 1$.

5.2 Dark Matter Candidates from Supersymmetry

Supersymmetry, a boson-fermion symmetry, has been invoked extensively as the solution of the so-called hierarchy problem. This problem is related to the fact that without some stabilizing influence radiative corrections in the electroweak theory would naturally push the Fermi scale v_F to have the value of whatever cutoff delimits the validity of the theory. Typically, this cutoff is imagined to be at the Planck scale M_P, and why $v_F << M_P$ is the hierarchy problem. This problem is resolved if there is some low energy (spontaneously broken) supersymmetry. Due to the fermion-boson nature of supersymmetry, radiative corrections of parameters in the electroweak theory (like v_F) are now only logarithmically dependent on the cutoff, not quadratically dependent. Hence, effectively, if there is some low energy supersymmetry one can contemplate having a hierarchy like $v_F << M_P$, because radiative shifts can only change v_F logarithmically.

In general, supersymmetric theories possess a discrete symmetry (R-symmetry) that distinguishes particles from their supersymmetric partners. As a result, the lightest supersymmetric particle (the LSP) is stable and, in principle, could be the source of the dark matter in the Universe. Indeed, it is known [64] that the energy density of particles of mass of $O(v_F)$, whose annihilation cross section is of electroweak strength, is of the order of the critical energy density that closes the Universe. With supersymmetric partners of ordinary particles having electroweak scale masses and interactions, the LSP is therefore an ideal candidate for the dark matter in the Universe [99]. In this review we will discuss both the cases of neutralinos (the SUSY partners of gauge and Higgs boson) and of gravitinos (the spin 3/2 partner of the graviton) as LSP candidates.

5.3 Extended Structures

Scalar fields are necessary ingredients of the standard electroweak model, as well as its supersymmetric extension. It is well known that theories with scalar fields can lead to the formation of nontopological solitons. These extended structures, known as Q-balls [100], may be stable or unstable and arise when some scalar field carries a conserved $U(1)$ charge. For example, in supersymmetric theories sleptons and squarks carry, respectively, lepton and baryon number.

In supersymmetric theories, more generally, Q-balls can develop along flat directions of the scalar potential [101]. These Q-balls can, in a number of instances, carry baryon number. If the baryon number of the Q-ball is large enough, and its mass is small enough, the baryonic Q-balls are stable. Because of their stability, one can imagine that these Q-balls could be the dark matter in the Universe.[18] Typically [101], if stable Q-balls exist they have both very large baryon number ($B \sim 10^{26}$) and are very massive ($M_Q \leq 10^{26}$ GeV). Unfortunately, this makes their detection very difficult, because their flux is very low [104].

5.4 Natural Connection of Axions with Leptogenesis

The scale of $U(1)_{PQ}$ breaking needed for axions to be the dark matter in the Universe ($f_a \simeq 0.3 \times 10^{12}$ GeV) is close enough to the mass of the lightest right handed neutrino ($M_1 \simeq 10^{10}$ GeV) needed for Leptogenesis to seek for a common linkage. In fact, the existence of such a linkage was observed long ago by Langacker, Peccei and Yanagida [105]. What these authors observed was that if M_1 were due to the VEV of a scalar field σ, one could identify this field as carrying a PQ-symmetry rather that lepton number.

Let us examine this assertion in a bit more detail by looking at the Yukawa interactions of the quarks and leptons with the three Higgs fields[19] ϕ_1, ϕ_2 and σ. Schematically, one has

$$\mathcal{L}_{\rm Yukawa} = h_\sigma \sigma N_R N_R + h \bar{N}_R \phi_2 \ell_L + h_u \bar{u}_R \phi_2 q_L + f_d \bar{d}_R \phi_1 q_L + f \bar{e}_R \phi_1 \ell_L + \text{h.c.}\,. \qquad (92)$$

One sees that Eq. (92) is invariant under a PQ-symmetry, where

$$\phi_1, \phi_2 \quad \rightarrow \quad e^{i\alpha}\phi_1, e^{i\alpha}\phi_2 \qquad (93)$$

$$N_R, l_R, u_R, d_R \quad \rightarrow \quad e^{i\alpha}N_R, e^{i\alpha}l_R, e^{i\alpha}u_R, e^{i\alpha}d_R, \qquad (94)$$

provided that

$$\sigma \rightarrow e^{-2i\alpha}\sigma. \qquad (95)$$

To allow $< \sigma >= f_a >> v_F$, as in all invisible axion models [106, 107], requires one fine tuning. In the above case, this requires the PQ-invariant term in the scalar potential

$$V = \kappa \sigma \phi_1 \phi_2 + \text{h.c.} \qquad (96)$$

[18]This, however, is not easily achieved because, in general, the squarks are unstable with their baryon number eventually residing on quarks. If the squarks are light enough, stability can be achieved. However, as Kasuya et al. point out [102], it is difficult to explain both the baryon asymmetry and the dark matter density simultaneously. Nevertheless, there are scenarios where unstable Q-balls are the source for both baryogenesis and neutralino dark matter [103].

[19]For a PQ symmetry to exist one needs to have two $SU(2)$ doublet Higgs fields, ϕ_1 and ϕ_2, rather than just the single Higgs field of the Standard Model H (and its Hermitian adjoint $H^\dagger$).

to have the constant $\kappa \sim v_F^2/f_a$, to allow electroweak symmetry breaking to occur at a scale much below the scale of $U(1)_{PQ}$ symmetry breaking ($v_F << f_a$).

The Yukawa couplings of this model guarantee that the mass of the lightest right-handed neutrino and f_a are related: $M_1 = 2(h_\sigma)_{11} f_a$. Thus, if axions are the source of the dark matter energy density in the Universe and the baryon aymmetry arises from Leptogenesis, because $\Omega_{\rm DM} \sim f_a$ and $\Omega_{\rm B} \sim M_1 \sim f_a$, their ratio is independent of the scale of $U_{PQ}(1)$ breaking. Hence it is perhaps not surprising that this ratio is of order unity.

5.5 The Gravitino Problem in Supersymmetric Theories

As we discussed in Section 4, for Leptogenesis to be effective, the mass of the lightest right-handed neutrino has to be greater than 2×10^9 GeV. This bound, in turn, means that thermal Leptogenesis must have occurred at temperatures above 2×10^9 GeV. Hence, if the Universe went through an inflationary period, as all evidence seems to suggest [4], the reheating temperature after inflation T_R must have been greater than 2×10^9 GeV for Leptogenesis to be the source of the matter-antimatter asymmetry in the Universe. This high reheating temperature is problematic for supersymmetric theories because it leads to an overproduction of light states, like the gravitino, with catastrophic consequences for the evolution of the Universe after inflation. Unless these observational inconsistencies can be avoided, it appears that Leptogenesis in supersymmetric theories cannot produce the desired baryon asymmetry in the Universe.

The production of gravitinos after inflation has been studied in some detail [108]. The thermal production of gravitinos produced by the strong interactions of quarks, squarks, gluons and gluinos is governed by the Boltzmann equation [109]

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = C_{3/2}(T), \tag{97}$$

where

$$C_{3/2}(T) = \frac{3\zeta(3)\alpha_3(T)}{\pi^2}\frac{T^6}{M_P^2}\left(1 + \frac{m_{\tilde{g}}^2}{3m_{3/2}^2}\right) F(T). \tag{98}$$

Here $F(T)$ is a thermal factor of O(10) and $m_{\tilde{g}}$ and $m_{3/2}$ are, respectively, the gluino and the gravitino masses. Integrating Eq. (97) to a reheating temperature T_R, the resulting relic density of produced gravitinos is given by

$$\Omega_{3/2}h^2 \simeq 0.44\alpha_3(T_R)\left[1 + \frac{1}{3}\left(\frac{\alpha_3(T_R)}{\alpha_3(\mu)}\right)^2\left(\frac{m_{\tilde{g}}(\mu)}{m_{3/2}}\right)^2\right]\left(\frac{T_R}{10^{10}\text{GeV}}\right)\left(\frac{m_{3/2}}{100\text{GeV}}\right), \tag{99}$$

where h is the scaled Hubble parameter and $\mu \sim M_Z$.

If gravitinos are stable (i.e. they are the LSP), the WMAP constraint on the amount of dark matter in the Universe [4]

$$\Omega_{DM}h^2 = 0.1126^{+0.0161}_{-0.0181} \tag{100}$$

constrains $\Omega_{3/2}h^2$ to be below this value and, for any given reheating temperature T_R and gravitino mass $m_{3/2}$, gives a bound on the gluino mass. If, on the other hand, the gravitinos are not stable, their rate of production for $T_R > 2 \times 10^9$ GeV is so large that subsequent gravitino decays completely alter the standard Big Bang Nucleosynthesis (BBN) scenario. Thus, in either case, there are severe constraints imposed on supersymmetric dark matter, which we will discuss in detail below.

If the gravitino is unstable, it has a long lifetime and decays during or after BBN for an interesting range of the gravitino mass, $m_{3/2} \simeq 100$ GeV $-$ 10 TeV. The gravitino decay products destroy the light elements produced by the BBN and hence the relic abundance of gravitinos is constrained from above to keep the success of the BBN [110]. This leads to an upper bound of the reheating temperature T_R after inflation, since the abundance of gravitinos is proportional to the reheating temperature. A recent detailed analysis derived a strigent upper bound $T_R < 10^{6-7}$ GeV when the gravitino decay has hadronic modes (see Fig. (5)) [90]. This upper bound is much lower than the temperature for Leptogenesis, $T_R > 2 \times 10^9$ GeV [69, 72]. Therefore, thermal Leptogenesis seems difficult to reconcile with low energy supersymmetry if gravitino masses lie in the range $m_{3/2} \simeq 100$ GeV $-$ 10 TeV - a natural range for Supergravity (SUGRA) models.

5.6 Solutions to the Gravitino Problem in Thermal Leptogenesis

There have been several attempts to solve the gravitino problem in thermal Leptogenesis. Here we will briefly review a number of these proposed solutions.

One possibility has been proposed by Pilaftsis who considers quasi-degenerate heavy Majorana neutrinos ($M_1 \simeq M_2$) [111]. In this model the lepton-asymmetry parameter ε is enhanced by a factor of $M_1/(M_1 - M_2)$ and hence the decays of both N_1 and N_2 may produce enough asymmetry even for $T_R < 10^{6-7}$ GeV. However, it is difficult to find a compelling justification for having such a degeneracy in the heavy neutrino spectrum.

Another proposal was made by Bolz, Buchmüller and Plümacher [112], who consider the case where the gravitino is the stable lightest SUSY particle (LSP). In this case the next lightest supersymmetric particle (NLSP) is the subject of the cosmological constraint, because its decay products may destroy the light elements created by the BBN, much like the unstable gravitino. Detailed analyses show that this scenario favors

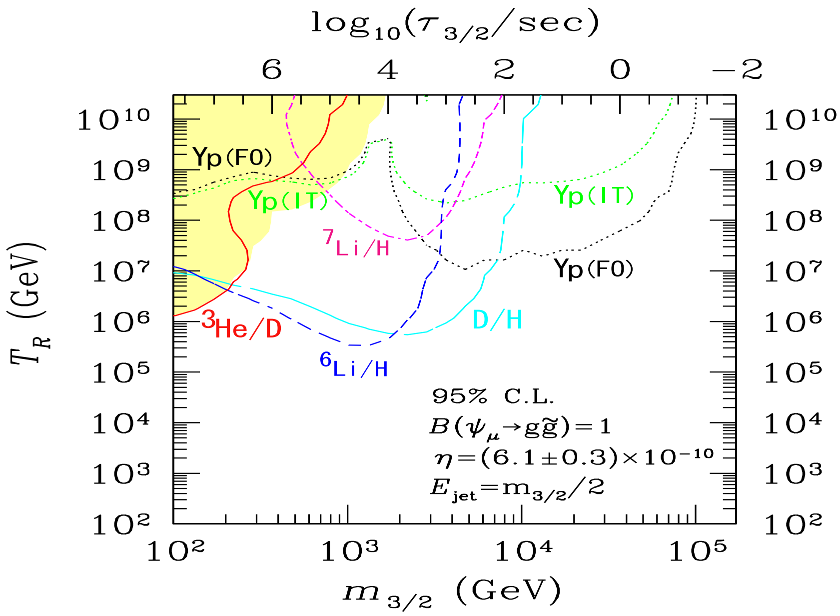

Figure 5: Upper bounds on the reheating temperature as function of the gravitino mass for the case where the gravitino dominantly decays into a gluon-gluino pair. From [90].

the $\tilde{\tau}$ NLSP compared to the neutralino NLSP and gravitino masses below $m_{3/2} \simeq$ 100 GeV [113, 114].[20] In general, the gravitino production can be dominated by NLSP decays [115] or by thermal processes [116].

A third proposed solution makes use of gauge-mediation model of supersymmetry breaking in which the gravitino is the stable LSP with a mass $m_{3/2} < 1$ GeV. It turns out that, if the gravitino mass is $m_{3/2} < 16$ eV [117], then there is no gravitino problem. However, in this case the gravitino cannot be the dark matter. It must be something else, perhaps the axion. For the range of gravitino masses $m_{3/2} \simeq 100$ keV $- 1$ GeV, there is the interesting possibility that late-time entropy production in a class of gauge mediation models can naturally make the gravitino the dominant component of dark matter [118]. In this scenario the reheating temperature can be as high as $T_R \simeq 10^{13}$ GeV. A light axino with mass $\mathcal{O}(1$ keV) as LSP and a gravitino as NLSP would solve the gravitino problem [119].

Finally, another possible solution arises if supersymmetry breaking effects are transmitted to the Standard Model sector through the scale anomaly, resulting in very heavy gravitino masses $m_{3/2} > 100$ TeV. In this case the gravitino decays before the time of BBN and hence there is no cosmological problem. However, the gravitino decay modes contain always one LSP and hence the relic abundance of the gravitino must be con-

[20] The gravitino mass is even less cnstrained if the LSP is a scalar neutrino or the gluino. For a class of supergravity models an upper bound of 5×10^9 GeV on the reheating temperature has been obtained [114].

strained from above so that the density of the nonthermal LSP produced by the gravitino decays does not exceed the dark-matter density. This condition leads to $T_R < 10^{11}$ GeV [120, 121], which is consistent with thermal Leptogenesis.

We stress here that each of the above 'solutions' predicts distinct particle spectra at the TeV scale, which may be testable in future collider experiments at the Large Hadron Collider (LHC). If one discovers supersymmetry but all the above possibilities are excluded experimentally, this would argue strongly against thermal Leptogenesis although nonthermal Leptogenesis could still be viable. On the other hand, if some of these scenarios are confirmed experimentally, thermal Leptogenesis will become much more compelling.

5.7 Leptogenesis and Lepton Flavor Violation in SUSY Models

Lepton flavor violation is another area in which thermal Leptogenesis and supersymmetry may have some linkages. If the neutrinos have a mass, lepton flavor violation (LFV) processes such as $\mu \to e + \gamma$ decay can occur. In non supersymmetric theories, these processes are strongly suppressed by a factor of $(m_\nu/M_W)^2$ in rate and hence are unmeasurable physically. However, this is not the case in the SUSY Standard Model, if the seesaw mechanism is effective.

In the SUSY Standard Model scalar quarks and leptons are assumed to have a universal SUSY-breaking soft mass, m_0, at the Planck scale. Otherwise one would have too large flavor-changing neutral currents (FCNC). However, even then quantum corrections resulting from Yukawa interactions of the quarks generate a violation of the universality of the soft masses for scalar quarks, which induces FCNC. In the lepton sector the Yukawa couplings of the superheavy Majorana neutrinos N_i also generates non-universal masses for scalar leptons that serves as a source of LFV [122].[21] If the relevant Yukawa couplings are of $O(1)$, or equivalently $M_3 \simeq 10^{15}$ GeV, one predicts a branching ratio for $\mu \to e + \gamma$ decay that may be testable in future experiments. However, an accurate prediction for LFV processes is very difficult, since it hinges on unkown Yukawa coupling constants [124]. In particular, the constraint coming from Leptogenesis that $M_1 \simeq 10^{10}$ GeV is not strong enough to suggest that LFV processes have potentially testable rates.

[21] The Yukawa couplings h_{ij} of the Higgs to the N_is and leptons induce flavor dependent soft masses for scalar leptons. At the one-loop level the induced mass is given by $(m_{\tilde{\ell}})^2_{ij} \simeq -6m_0^2/(4\pi)^2 h^\dagger_{ik} h_{kj} ln(M_P/M_k)$, where m_0 is the universal soft mass for scalar leptons at the Planck scale M_P. Thus, one may obtain information on the high-energy Yukawa coupling h_{ij} and the heavy neutrino masses M_k by measuring directly the mass matrix for the scalar leptons. However, the phases in $h^\dagger h$ are different from the phases contributing to Leptogenesis [123].

6 Nonthermal Leptogenesis

Supersymmetry is an important symmetry for the unification of all interactions and all matter, and the SUSY Standard Model is considered as a plausible scenario for producing new physics at the TeV scale. Thus, it is quite interesting to consider theories where supersymmetry is spontaneously broken in a hidden sector connected to ordinary matter by gravitational strength interactions– the SUGRA framework. The seesaw mechanism is easily incorporated into this framework. However, as we discussed in some detail in Section 5, the gravitino problem argues against thermal Leptogenesis, particularly in SUGRA.

A possible solution to this problem may be provided by nonthermal Leptogenesis [10,125–129], where one does not have a strong constraint on the reheating temperature. We will discuss here specifically nonthermal Leptogenesis via inflaton decay [125,126], which we consider an interesting scenario. In the next subsection, we present general arguments for this scenario and show that it suggests a lower bound on the mass of the heaviest light neutrino $m_3 > 0.01$ eV. In the subsequent subsection, we will also discuss the Affleck-Dine mechanism [10] for Leptogenesis which, specifically in supersymmetric theories, is also an interesting mechanism to generate the matter-antimatter asymmetry.

6.1 Nonthermal Leptogenesis via Inflaton Decay

Inflation early on in the history of the Universe is one of the most attractive hypothesis in modern cosmology, because it not only solves long-standing problems in cosmology, like the horizon and the flatness problems [130], but also accounts for the origin of density fluctuations [131]. In this subsection we discuss the hypothesis that the inflaton Φ decays dominantly into a pair of the lightest heavy Majorana neutrinos, $\Phi \to N_1 + N_1$. We assume, for simplicity, that other decay modes including those into pairs of N_2 and N_3 are energetically forbidden. The produced N_1 neutrinos decay subsequently into $H + \ell_L$ or $H^\dagger + \ell_L^\dagger$. If the reheating temperature T_R is lower than the mass M_1 of the heavy neutrino N_1, then the out-of-equilibrium condition [11] is automatically satisfied.

The above two channels for N_1 decay have different branching ratios when CP conservation is violated. Interference between tree-level and one-loop diagrams generates a lepton-number asymmetry [57]. Following our discussion in Section 4, the lepton asymmetry parameter ε can be written as [128,129,132] [22]

$$\varepsilon = -\frac{3}{8\pi}\frac{M_1}{\langle H\rangle^2} m_3 \delta_{\rm eff} \,, \tag{101}$$

[22] Because of supersymmetry, the asymmetry parameter ε below is a factor of 2 larger than that given in Eq. (58).

where the effective CP-violating phase $\delta_{\rm eff}$ is given by

$$\delta_{\rm eff} = \frac{{\rm Im}\left[h_{13}^2 + \frac{m_2}{m_3}h_{12}^2 + \frac{m_1}{m_3}h_{11}^2\right]}{|h_{13}|^2 + |h_{12}|^2 + |h_{11}|^2} \ . \tag{102}$$

Numerically, one obtains for the ε parameter

$$\varepsilon \simeq -2 \times 10^{-6} \left(\frac{M_1}{10^{10}{\rm GeV}}\right)\left(\frac{m_3}{0.05{\rm eV}}\right)\delta_{\rm eff} \ . \tag{103}$$

The chain decays $\Phi \to N_1 + N_1$ and $N_1 \to H + \ell_L$ or $H^\dagger + \ell_L^\dagger$ reheat the Universe producing not only the lepton-number asymmetry but also entropy for the thermal bath. The ratio of the lepton number to entropy density after reheating is estimated to be [126]

$$\begin{aligned} \frac{n_L}{s} &\simeq -\frac{3}{2}\varepsilon\frac{T_R}{m_\Phi} \\ &\simeq 3 \times 10^{-10}\left(\frac{T_R}{10^6{\rm GeV}}\right)\left(\frac{M_1}{m_\Phi}\right)\left(\frac{m_3}{0.05{\rm eV}}\right), \end{aligned} \tag{104}$$

where m_Φ is the inflaton mass and we have taken $\delta_{\rm eff} = 1$. This lepton-number asymmetry is converted into a baryon-number asymmetry through the sphaleron effects and one obtains [35]

$$\frac{n_B}{s} \simeq -\frac{8}{23}\frac{n_L}{s} \ . \tag{105}$$

We should stress, here, an important merit of the inflaton-decay scenario: It does not require a reheating temperature $T_R \sim M_1$, but it requires only $m_\Phi > 2M_1$. On the other hand, for thermal Leptogenesis to work it is necessary that $T_R \sim M_1$, which necessitates higher reheating temperature for Leptogenesis to produce enough matter-antimatter asymmetry.

If one assumes that $T_R < 10^7$ GeV to satisfy the cosmological constraint on the gravitino abundance [90] discussed earlier and uses $m_\Phi > 2M_1$, the observed baryon number to entropy ratio [4] gives a constraint on the heaviest light neutrino:

$$m_3 > 0.01 \ {\rm eV}. \tag{106}$$

It is very interesting that the neutrino mass suggested by atmospheric neutrino oscillation experiments, $\sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$ eV, just satisfies the above constraint. However, to get this bound we assumed that the inflaton decays dominantly into a pair of N_1s. If this branching ratio is only 10 %, the lower bound on the neutrino mass exceeds the observed neutrino mass $\sqrt{\Delta m^2_{\rm atm}} \simeq 0.05$ eV.

A variety of models have been considered to restore the bound of Eq. (106) by imposing a symmetry. However, it is perhaps most interesting to consider that the scalar partner of the heavy Majorana neutrino N_1 is the inflaton itself [127], and the inflaton decay into a lepton plus a Higgs boson gives an effective branching ration of 100%. In this model, one must assume that the initial value of the scalar partner of N_1 is much larger than the Planck scale to cause inflation (chaotic inflation [133]). However, chaotic inflation is not easily realized in SUGRA, because the minimal supergravity potential has an exponential factor, $\exp(\phi^*\phi/M_G^2)$, that prevents any scalar field ϕ from having a value larger than the reduced Planck scale $M_G \simeq 2.4 \times 10^{18}$ GeV. Ref. [134] uses a restricted form of the Kahler potential.

6.2 Affleck-Dine Leptogenesis

In the SUSY Standard Model, in the limit of unbroken supersymmetry, some combinations of scalar fields do not enter the potential, constituting so-called flat directions of the potential. Since the potential is almost independent of these fields, they may have large initial values in the early Universe. Such flat directions receive soft masses in the SUSY-breaking vacuum. When the expansion rate $H_{\rm exp}$ of the Universe becomes comparable to their masses, the flat directions begin to oscillate around the minimum of the potential. If the flat directions are made of scalar quarks and carry baryon number, the baryon-number asymmetry can be created during these coherent oscillations. This is the Affleck-Dine (AD) mechanism for Baryogenesis [10].

QCD corrections, however, make the potential of the AD fields milder than $|\phi|^2$. This allows non-topological soliton solutions (Q-balls) [135] to form in the early Universe, as a result of the coherent oscillations in the flat directions. Because Q-balls have long lifetimes, their decays produce a huge amount of entropy at late times. To avoid this problem one must choose parameters in the SUSY theory so that the density of the lightest SUSY particle (LSP) does not exceed the dark matter density in the present Universe [135]. Although this may not be a problem, it is much safer to consider flat directions without QCD interactions, because such directions most likely do not have Q-ball solutions.

The most interesting candidate [128] for such a flat direction is

$$\phi_i = (2H\ell_i)^{1/2}, \qquad (107)$$

where ℓ_i is the lepton doublet field of the *i-th* family. Here, H and ℓ_i represent the scalar components of the corresponding chiral multiplets. The Yukawa interactions of H make the potential of ϕ_i steeper than the mass term and hence there is no instability of the coherent oscillation (i.e. there are no Q-ball solutions). Because this flat direction carries lepton number, a lepton asymmetry will be created during the coherent oscillation (AD Leptogenesis) [128]. Sphaleron processes then transmute, in the usual fashion, this lepton asymmetry into a baryon asymmetry.

The seesaw mechanism induces a dimension-five operator in the superpotential for the theory,[23]

$$W = \frac{m_\nu}{2|\langle H\rangle|^2}(\ell H)^2, \tag{108}$$

where we have used a basis in which the neutrino mass matrix is diagonal. With this superpotential we have a SUSY-invariant potential for the flat direction ϕ given by

$$V_{\rm SUSY} = \frac{m_\nu^2}{4|\langle H\rangle|^4}|\phi|^6. \tag{109}$$

In addition to the SUSY-invariant potential we have a SUSY-breaking potential,

$$\delta V = m_\phi^2|\phi|^2 + \frac{m_{\rm SUSY} m_\nu}{8|\langle H\rangle|^2}(a_m\phi^4 + {\rm h.c.}). \tag{110}$$

Here, a_m is a complex number. We take $m_\phi \simeq m_{\rm SUSY} \simeq 1$ TeV and $|a_m| \sim 1$. The second term in δV is very important, because it gives rise to the lepton-number generation.

We assume that the flat direction ϕ acquires a negative (mass)^2 induced by the inflaton potential and rolls down to the point balanced by the SUSY-invariant potential $V_{\rm SUSY}$ during inflation. Thus, the AD field ϕ has an initial value of $\sqrt{H_{\rm inf}|\langle H\rangle|^2/m_\nu}$, where $H_{\rm inf}$ is the Hubble constant (the expansion rate) during inflation. ϕ decreases in amplitude gradually after inflation, and begins to oscillate around the potential minimum when the Hubble constant $H_{\rm exp}$ of the Universe becomes comparable to the SUSY-breaking mass m_ϕ. At the beginning of the oscillation, the AD field has the value $|\phi_0| \simeq \sqrt{m_\phi|\langle H\rangle|^2/m_\nu}$ which, as shown below, is an effective initial value for Leptogenesis.

Let us consider now lepton-number generation in this scenario. The evolution of the AD field ϕ is described by

$$\frac{\partial^2\phi}{\partial t^2} + 3H_{\rm exp}\frac{\partial\phi}{\partial t} + \frac{\partial V}{\partial\phi^*} = 0\ , \tag{111}$$

where $V = V_{\rm SUSY} + \delta V$. Because the lepton number is given by

$$n_{\rm L} = i\left(\frac{\partial\phi^*}{\partial t}\phi - \phi^*\frac{\partial\phi}{\partial t}\right)\ , \tag{112}$$

the evolution of $n_{\rm L}$ is given by

$$\frac{\partial n_{\rm L}}{\partial t} + 3H_{\rm exp}n_{\rm L} = \frac{m_{\rm SUSY}m_\nu}{2|\langle H\rangle|^2}{\rm Im}(a_m^*\phi^{*4})\ . \tag{113}$$

The motion of ϕ in the phase direction generates the lepton number. This is predominantly created just after the AD field ϕ starts its coherent oscillation, at a time

[23]For ease of notation we have dropped the subscript i below.

$t_{\rm osc} \simeq 1/H_{\rm osc} \simeq 1/m_\phi$, because the amplitude $|\phi|$ damps as t^{-1} during the oscillation. Thus, we obtain for the lepton number

$$n_{\rm L} \simeq \frac{m_{\rm SUSY} m_\nu}{2|\langle H\rangle|^2}\delta_{\rm eff}|a_m\phi_0^4| \times t_{\rm osc}\ , \tag{114}$$

where $\delta_{\rm eff} = \sin(4\arg\phi + \arg a_m)$ represents an effective CP-violating phase. Using $m_{\rm SUSY} \simeq m_\phi$, $|\phi_0| \simeq \sqrt{m_\phi|\langle H\rangle|^2/m_\nu}$ and $t_{\rm osc} \simeq 1/m_\phi$, we find

$$n_{\rm L} \simeq \delta_{\rm eff} m_\phi^2 \frac{|\langle H\rangle|^2}{2m_\nu}\ . \tag{115}$$

After the end of inflation, the inflaton begins to oscillate around the potential minimum and $n_{\rm L}/\rho_{\rm inf}$ stays constant until the inflaton decays. Here $\rho_{\rm inf}$ is the energy density of the inflaton. The inflaton decay reheats the Universe producing entropy s. Because $\rho/s = 2T_R/4$, we find for the lepton-number asymmetry the expression

$$\frac{n_{\rm L}}{s} \simeq \left(\frac{\rho_{\rm inf}}{s}\right)\left(\frac{n_{\rm L}}{\rho_{\rm inf}}\right) \simeq \delta_{\rm eff}\frac{3T_R}{4M_G}\frac{|\langle H\rangle|^2}{6m_\nu M_G}\ . \tag{116}$$

Here we have used $\rho_{\rm inf} \simeq 3m_\phi^2 M_G^2$ at the begining of the AD field oscillation (when most of the lepton number is generated). This lepton-number asymmetry is converted to a baryon-number asymmetry by the KRS mechanism. In this way one obtains for the baryon-number asymmetry

$$\frac{n_{\rm B}}{s} \simeq \frac{1}{23}\frac{|\langle H\rangle|^2 T_R}{m_\nu M_G^2}\ . \tag{117}$$

The observed ratio $n_{\rm B}/s \simeq 0.9\times 10^{-10}$ implies $m_\nu \simeq 10^{-9}$ eV for $T_R \simeq 10^6$ GeV. This small mass corresponds to the mass of the lightest neutrino. We should note that for such a low reheating temperature one may neglect the effects due to thermal mass for the AD field ϕ [136].

7 Conclusions and Summary of Results

In this article we have discussed the physical mechanism responsible for the origin of matter in the Universe. Both the rather large observed value for the ratio of baryons to photons, η_B, in the present epoch and the absence of antimatter are the consequences of a primordial asymmetry between matter and antimatter generated early on in the Universe. Although a variety of mechanisms have been proposed for producing this primordial asymmetry, in this review we have focused on Leptogenesis as the origin of matter. In our view, this is the most appealing scenario for the origin of matter, for at least three reasons:

1) Explicit lepton number violation is very natural once one includes right-handed neutrinos in the Standard Model. Furthermore, the lightness of the observed neutrinos strongly suggests, through the seesaw mechanism, the presence of superheavy neutrinos, whose decays can produce a lepton-antilepton asymmetry.

2) Quantum mechanically, through the KRS mechanism, one can automatically turn a leptonic asymmetry into a baryonic asymmetry. Indeed, because of the existence of these sphaleron processes, the origin of matter is linked to phenomena in the early Universe that result in the establishment of a (B-L)-asymmetry, like Leptogenesis.

3) If neutrino masses lie in the range $10^{-3}\mathrm{eV} < \mathrm{m_i} < 0.1\mathrm{eV}$, as suggested by neutrino oscillation experiments, the leptonic asymmetry produced in thermal Leptogenesis is both independent of the abundance of heavy neutrinos and of any pre-existing asymmetry and has the right magnitude to yield the observed value for η_B.

Because, in the final analysis, η_B is just one number, it is important to ask if the particular mechanism proposed for the origin of matter has other consequences. Thus in this review we examined in some detail how Leptogenesis fit with ideas proposed to explain the dark matter that constitutes about 25% of the Universe's energy density.[24] We pointed out that axionic dark matter is perfectly compatible with Leptogenesis. Indeed, it is possible to very naturally link the scale of the heavy neutrinos with that of $U(1)_{PQ}$ breaking f_a, so that the ratio $\Omega_{\mathrm{DM}}/\Omega_B$ is independent of these large scales. The situation, however, is more complex in the case of supersymmetric dark matter.

To be effective, thermal Leptogenesis needs to occur at high temperatures, above $T = 2 \times 10^9$ GeV. This means that the Universe after inflation must have reheated to at least this temperature. However, in supersymmetric theories such a high reheating temperature is problematic as it leads to an overproduction of gravitinos. When they decay, gravitinos of such abundances completely alter the primordial abundance of light elements produced in Big Bang Nucleosynthesis. The gravitino problem, however, is not fatal as there are a number of ways to mitigate the overproduction of gravitinos. Nevertheless, if Leptogenesis is at the root of the origin of matter, the supersymmetric spectrum at low energies and the nature of the LSP are quite constrained. Thus, in a sense, Leptogenesis is also quite predictive in this context.

Although much of our review, very naturally, focused on thermal Leptogenesis, we also discussed two examples where matter originated through a leptonic asymmetry produced in nonthermal processes. These models, although much more speculative, illustrate some of the possible other options for the origin of matter. Naturally, in this case some of the specific predictivity is lost.

[24]We did not try to examine models of dark energy in the light of Leptogenesis, because our understanding of dark energy is still in its infancy.

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