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The staggered charge-order phase of the extended Hubbard model in the atomic limit

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We study the phase diagram of the extended Hubbard model in the atomic limit. At zero temperature, the phase diagram decomposes into six regions: three with homogeneous phases (characterized by particle densities $\rho = 0$, 1, and 2 and staggered charge density $\Delta = 0$) and three with staggered phases (characterized by the densities $\rho = \frac{1}{2}$, 1, and $\frac{3}{2}$ and staggered densities $|\Delta| = \frac{1}{2}$, 1, and $\frac{1}{2}$). Here we use Pirogov-Sinai theory to analyze the details of the phase diagram of this model at low temperatures. In particular, we show that for any sufficiently low nonzero temperature the three staggered regions merge into one staggered region S, without any phase transitions (analytic free energy and staggered order parameter Δ) within S.

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Estimates of semiinvariants for the Ising model at low temperatures

R.L. Dobrushin [1 2]

Let $V \subset \mathbb{Z}^d$ be a finite subset of d-dimensional integer lattice $\mathbb{Z}^d$ and $X(V) = \{-1,1\}^V$ be the set of configurations $x = (x_t, t \in V)$, where $x_t = \pm 1$. The (symmetric ferromagnet) Ising distribution in the volume V with the plus-boundary condition and a converse temperature $\beta > 0$ is the probability distribution on the set $X(V)$ such that the probability

(1) $$p_V(x) = (Z(V))^{-1} \exp\{-\beta U_V(x)\}, \quad x \in X(V),$$

where the partition function

(2) $$Z(V) = \sum_{x \in X(V)} \exp\{-\beta U_V(x)\}$$

and the energy

(3)
$$U_V(x) = -\left(\sum_{\{s,t\} \subset V : |s-t|=1} x_s x_t + \sum_{\{s,t\}: t \in V, s \in V^c, |s-t|=1} x_t \right), \quad x \in X(V).$$

Let $T \subset V$ be a finite set. The generating function of the values of the field on the set T is defined by the relation

(4) $$F_T(V; z_t, t \in T) = \sum_{x \in X(V)} p_V(x) \exp\left\{ \sum_{t \in T} z_t x_t \right\},$$

where $z_t, t \in T$, are complex numbers. Let $r = (r_t, t \in T)$ be a function with integer values $r_t \geq 1, t \in T$, and $|r| = \sum_{t \in T} r_t$. The semiinvariant (in other terminologies cumulant, truncated correlation function) of order r of the values of the field on the set T is defined as the value of the partial derivate

(5) $$s_V(T, r) = \left. \frac{\partial^{|r|} \ln F_T(V; z_t, t \in T)}{\prod_{t \in T} \partial z_t^{r_t}} \right|_{z_t \equiv 0, t \in T,}.$$

We introduce a geometrical characteristic of the set T

(6) $$q(T) = \max_{S \subseteq T} \frac{|S|}{(\text{diam } S)^{d-1}}$$

which we call the concentration coefficient of the set T. (Here $|A|$ is the number of elements in a finite set A.) This coefficient do not exceed $T^{\frac{1}{d}}$ and takes values of this order for sets like cubes or spheres but can be arbitrary small for sets evenly spread on the lattice.

The following estimate is the main result of the paper.

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Theorem. *For each $d \geq 2$ there exist constants $\tilde{\beta} = \tilde{\beta}_d > 0$ and $K = K_d < \infty$ such that for all $\beta > \tilde{\beta}$, all finite sets $T \subset V \subset \mathbb{Z}^d$, where $|T| > 1$, and all $r = (r_t, t \in T)$ the following estimates for the semiinvariants hold*

$$|s_V(T, r)| \leq (K \max(q(T), 1))^{|r|} \exp\{-2(\beta - \tilde{\beta})|\Gamma_T|\}, \tag{7}$$

where $|\Gamma_T|$ is the number of bounds in the minimal contour containing all the set T.

The estimate (7) is essentially stronger than the estimates which were obtained earlier. The multiplier in (7) which exponentially depend on $|\Gamma_T|$ seems natural since it estimates the probability that the set T lies inside a contour arounding a piece of another phase. The improvement in the comparison with the previous estimates is connected with the preexponential multiplier which waslarger than $2^{2^{|r|}}$ in the previous estimates.

In the derivation of the formulated theorem we use a new variant of the cluster expansion method and instead of a direct application of these expansions to contours we apply them to some more complex objects adequate to the considered problem. In a difference with the other approaches we apply elementary facts of the theory of analytical functions instead of involved combinatorial estimates for terms of cluster expansion and it simplifies the construction and gives estimates convenient for applications.

Fluctuations of Shape of Phase Boundaries in the 2D Ising Ferromagnet

R.Dobrushin [1,2] and O.Hryniv [1,3]

Let V_N denote the vertical strip consisting of the points from the integer lattice $\mathbb{Z}^2$, $V_N = \{t = (t_1, t_2) \in \mathbb{Z}^2 : 0 < t_1 < N\}$. Fix any real number $\varphi \in (-\pi/2, \pi/2)$ and consider two-component boundary condition $\overline{\sigma}_t^\varphi$, $t \in \mathbb{Z}^2$, $\overline{\sigma}_t^\varphi = 1$, if $t_2 > t_1 \tan\varphi$, and $\overline{\sigma}_t^\varphi = -1$ otherwise. Then the boundary $\Gamma(\sigma)$ of any configuration $\sigma = \sigma_{V_N} \in \{-1, +1\}^{V_N}$ under the boundary conditions $\overline{\sigma}^\varphi$ contains exactly one open contour S to be called below the phase boundary. Clearly, this contour connects the points $(0, 1/2)$ and $(N, [N\tan\varphi] + 1/2)$. For the fixed value φ we denote by $\mathcal{T}_N^\varphi$ the collection of all phase boundaries in V_N compatible with the boundary condition $\overline{\sigma}^\varphi$.

We define the probability distribution $\mathrm{P}_{N,\beta,\varphi}(\cdot)$ in the set $\mathcal{T}_N^\varphi$ as the distribution induced by the "Ising ferromagnet distribution" in the vertical strip V_N. More precisely, for ant $M > 1$ we denote $V_{N,M} = \{t = (t_1, t_2) \in \mathbb{Z}^2 : 0 \le t_1 \le N, 1 - M \le t_2 \le M\}$ and consider the usual Ising ferromagnet distribution in $V_{N,M}$ corresponding to the hamiltonian

$$\mathcal{H}(\sigma|\overline{\sigma}) = -\sum_{\substack{s,t \in V_{N,M} \\ |t-s|=1}} \sigma_s \sigma_t - \sum_{\substack{s \in V_{N,M}, t \in \mathbb{Z}^2 \setminus V_{N,M} \\ |t-s|=1}} \sigma_s \overline{\sigma}_t$$

and the inverse temperature $\beta > 0$. Let $\mathrm{P}_{N,M,\beta,\varphi}(\cdot)$, $M > N\tan\varphi$, denote the induced distribution in the space $\mathcal{T}_{N,M}^\varphi$ of the phase boundaries in $V_{N,M}$ compatible with the boundary conditions $\overline{\sigma}^\varphi$. It can be shown that for all sufficiently large β, $\beta \ge \beta_0 > 0$, the limiting distribution $\mathrm{P}_{N,\infty,\beta,\varphi}(\cdot)$ makes sense as a distribution in $\mathcal{T}_N^\varphi$ and we denote it by $\mathrm{P}_{N,\beta,\varphi}(\cdot)$. Let $F(H)$ denote the free energy corresponding to the grand canonical Gibbs distribution for the height $h = [N\tan\varphi]$ of the phase boundary $S \in \mathcal{T}_N^\varphi$.

Every phase boundary $S \in \mathcal{T}_N^\varphi$ is contained in a certain rectangle $[0, N] \times [1 - M, M]$ if only M is sufficiently large. Then the polygon S divides this rectangle into two parts, "upper" and "lower" ones, with the areas Q_N^+ and Q_N^- correspondingly. Consider the quantity

$$a(S) = a_N(S) \equiv \frac{1}{2}\big(Q_N^- - Q_N^+\big) \tag{1}$$

to be called below the area under the contour S. Clearly, this definition does not depend on M if only M is sufficiently large, $M \ge M_0(S)$.

It follows from the definition (1) that the variable $a(S)$ attains only half-integer values. For the future references we fix a sequence of numbers $N^2 q_N$ such that

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$q_N - q = o(N^{-1/2})$, $q \geq 0$, as $n \to \infty$, which satisfies for all N the following condition

$$\mathrm{P}_{N,\beta,\varphi}(a_N(S) = N^2 q_N) > 0. \tag{2}$$

Fix some number N and any contour $S \in \mathcal{T}_N^{\varphi}$. Then for every natural number $k = 1, \ldots, N$ we define $g_N^+(k) = \max\{t_2 : (k, t_2) \in S\}$ and consider the random polygonal function $\xi_N^+(t)$, $t \in [0,1]$, (with $g_N^+(0) = \frac{1}{2}$)

$$\xi_N^+(t) \equiv g_N^+([Nt]) + \frac{\{Nt\}}{N}\big(g_N^+([Nt+1]) - g_N^+([Nt])\big).$$

Here $[x]$ denotes the integral part of the real number x and $\{x\}$ denotes its fractional part. The random function $\xi_N^+(t)$ presents the linear interpolation of the sequence $g_N^+(k)$, $k = 0, \ldots, N$.

In view of the condition (2) the random process $\theta_N(t) = \big(\xi_N^+(t) \big| \, a_N(S) = N^2 q_N\big)$ is well-defined and we denote its distribution in the space $\mathbf{C}[0,1]$ of continuous functions on the segment $[0,1]$ by μ_N.

Determine $H_0 = H_0(q,\varphi)$, $H_1 = H_1(q,\varphi)$ from the equations

$$\frac{1}{\beta}\int_0^1 x\, F'(H_1 + H_0 x)\, dx = q,$$

$$\frac{1}{\beta}\int_0^1 F'(H_1 + H_0 x)\, dx = \tan\varphi,$$

where $F(\cdot)$ denotes the free energy, define

$$\mathbf{e}_{H_0,H_1}(t) \equiv \left(F(H_1 + H_0) - F(H_1 + H_0 - H_0 t)\right)/H_0\beta$$

and consider normalized fluctuations of paths $\theta_N(t)$ around $N\mathbf{e}_{H_0,H_1}(t)$,

$$\theta_N^*(t) \equiv \frac{1}{\sqrt{N}}\big(\theta_N(t) - N\mathbf{e}_{H_0,H_1}(t)\big).$$

Let μ_N^* be the corresponding measure in $\mathbf{C}[0,1]$.

Theorem. *Let a sequence q_N be as described above.*

Then the sequence of measures μ_N^* converges weakly to some Gaussian measure μ^* in $\mathbf{C}[0,1]$. The limiting measure μ^* coincides with the conditional distribution of the process

$$\overline{\xi}(t) \equiv \frac{1}{\beta}\int_0^t \left(F''(H_1 + H_0 - H_0 x)\right)^{1/2} dw_x, \qquad t \in [0,1],$$

where dw_x refers to the white noise, conditioned by the condition

$$\overline{\eta} \equiv \int_0^1 \overline{\xi}(t)\, dt = 0.$$

Ill-defined renormalization group maps: some new results

A.C.D. van Enter [1]

We show how in two examples applying a renormalization group map to a Gibbsian measure in the uniqueness regime results in a non-Gibbsian measure. This means that defining the implementation of an RG map from Hamiltonians to Hamiltonians is an ill-posed problem.

The two examples are:

A. Decimation, applied to the high-q Potts model above the transition temperature.

B. Majority rule on odd blocks, applied to the high field, low temperature Ising model.

Our scheme of proof is based upon the fundamental ideas of Griffiths, Pearce and Israel. We need to find a configuration of the block spins (= renormalized spins), which, acting as a constraint, is such, that the thus constrained system has a phase transition. This, by some technical steps, then can be shown to imply the violation of the "quasilocality" property for the renormalized measure. Quasilocality means that *any* influence from far away has to be transmitted via intermediate regions, and cannot be a direct influence. For finite discrete spins, like Potts or Ising spins, the quasilocality property (also known as "almost Markov" property) is equivalent to being a Gibbs measure for a reasonable interaction ("Hamiltonian"). Thus the existence of long-range order in the internal variables (the variables to be integrated out) implies non-(quasi)locality in the block spin variables. In the case of the decimated Potts model, the "constraint configuration" is the one obtained by taking all Potts spins on the decimated sublattice to be the same, say in state 1 (out of q possibles). This enhances the long-range order in the 1-direction, and as a consequence the transition temperature of the first-order order-disorder transition is raised by the constraint. At the transition temperature, the constrained system can be either in the disordered or in the 1-ordered state. A corollary of this result is that we obtain the first example where the Dobrushin-Shlosman Complete Analyticity condition is violated *above* the transition temperature.

In the example of the majority rule map for the low temperature, strong field Ising model, the "constraint configuration" is obtained by taking the majority in each block opposing the magnetic field. Then, as the majority in each block will tend to be small, due to the magnetic field, about half of the spins will be plus and half of the spins will be minus in each block; there will be an interface in each block separating the pluses from the minuses. Matching these interfaces gives rise to the ground states, c.q. low temperature states. The phase transition in the constrained system describes a periodic long-range order of (in two dimensions) either horizontal or vertical alternating strips of pluses and minuses. Because for both horizontal and vertical strip states there are two possibilities, the total number of Gibbs measures for the constrained system is four. Just as in the case of block-averaging , the RG pathologies enter into the region of Complete Analyticity. The proofs of the phase transitions in the constrained systems, which are the main new elements in our work, can be done either by Reflection Positivity methods , or by the Bricmont-Kuroda-Lebowitz generalization of Pirogov-Sinai theory.

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Percolation Techniques in Disordered Spin Systems

Guy Gielis [1] and Christian Maes [2]

We consider lattice spin systems with short range but random and unbounded interactions. We give an elementary proof of uniqueness of Gibbs measures at high temperature or strong external fields, and of the exponential decay of the corresponding quenched correlation functions. We give criteria for ergodicity of spin flip dynamics and estimate the speed of convergence to the unique invariant measure. We find for this convergence a stretched exponential in time for a class of directed dynamics such as in the disordered Toom and Stavskaya models. For the general case, we show that the relaxation is faster than any power in time. All results are obtained via domination arguments with corresponding percolation problems in disordered media.

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Stratified Low Temperature Phases of Ising Type Models. A General Pirogov-Sinai Approach

Petr Holický [1] and Miloš Zahradník [2]

The keywords of our exposition are :"contour"(either a classical Peierls contour or a "wall" in the terminology of R.L.Dobrushin) , "expansion" (also a *partial* expansion as an intermediate step of the construction),and "free energy density" (of a "metastable" submodel).

We adapt and improve the existing Pirogov-Sinai technology thus obtaining a general and unifying approach to the study of low temperature phases for classical spin models whose hamiltonian may even not to be translation invariant but is "stratified" in the sense that it depends on only one coordinate of the lattice. Examples are "stratified" versions of classical models like the Ising model with "vertically dependent" external field; models in halfspaces or layers (like the -so called-Basuev states)and also those ordinary translation invariant models where phases with rigid interfaces (one or more) appear.The latter examples include wetting phenomena in models of Blume Capel type or in the Potts model -with a large number of spins-below the critical temperature.

Our method brings some simplification and sharpening even to the ordinary Pirogov-
-Sinai theory and overcomes the previous approaches of the authors.

Our main result transcripts the question of characterizing all stratified *Gibbs states* of the given model to the question of finding all *ground states* of some auxilliary *one dimensional* model. This auxilliary one dimensional model is again of the Ising type; with the additional "external field"(more adequately "cluster field") being given by an appropriate *expansion* of the original model;the cluster expansion terms are in principle computable and their quick convergence is established.

These expansions(of submodels which are called "metastable" by us) are the key construction of the paper.They represent our new approach to the Pirogov Sinai theory where the concept of a "contour functional" still remains as an important *testing quantity* but the notion of a contour model disappeared completely.The cluster expansions used by us are constructed *successively*(in an infinite sequence of *partial* expansions) together with the successive clarification what the "metastable "model should be.These expansions enter the other basic constructions of the paper and cannot be viewed merely as a suitable method to estimate the partition functions. .They are based on a repeated use of one cluster expansion step - called a "recoloring" (or an expansion) of a contour by us- and they are completely self-contained.

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Conditional Limit Theorems and Equivalence of Ensembles

J.T. Lewis [1] , C.-E. Pfister [2] and W.G. Sullivan[1, 3]

We prove several conditional limit theorems related to the problem of equivalence of ensembles of Statistical Mechanics in *Entropy, Concentration of Probability and Conditional Limit Theorems* (DIAS–preprint). In particular we prove the following result. For each $j \in \mathbb{Z}^d$, let $(\Omega_j, \mathcal{F}_j, \lambda_j)$ be a copy of the probability space $(S, \mathcal{S}, \beta)$, where the measurable space $(S, \mathcal{S})$ is a standard Borel space. We denote by $(\Omega, \mathcal{F}, \lambda)$ the product probability space. On the space $\mathcal{M}_1^+$ of probability measures on $(\Omega, \mathcal{F})$ we put the topology of local convergence, ν_n converges to ν iff for each quasilocal function f, $\lim_n \nu(f) = \nu(f)$. Let f be a real–valued quasilocal function; we define

$$p(f) := \lim_n \frac{1}{|\Lambda_n|} \ln \int_\Omega \exp\{\sum_{j\in\Lambda_n} f(\theta_j \omega)\}\lambda[d\omega];$$

here and below θ_j is the action of the translation by j, $\Lambda_n := [-n, n]^d \cap \mathbb{Z}^d$ and $|\Lambda_n|$ is the cardinality of the set Λ_n. We recall that a translation invariant probability measure α is a *f–equilibrium state* iff

$$\int_\Omega f(\omega)\alpha[d\omega] = p(f) + h(\alpha|\lambda),$$

where $h(\alpha|\lambda)$ is the specific relative entropy of α with respect to λ.

Let us fix a $\mathbb{R}^k$–valued quasilocal function $\boldsymbol{g} : \Omega \mapsto \mathbb{R}^k$. We define a random variable $T_n : \Omega \mapsto \mathbb{R}^k$, $n \in \mathbb{N}$,

$$T_n(\omega) := \frac{1}{|\Lambda_n|} \sum_{j\in\Lambda_n} \boldsymbol{g}(\theta_j \omega);$$

a conditioned measure (microcanonical measure) $\lambda[\,\cdot\,|\{T_n \in C\}]$, $n \in \mathbb{N}$ and $C \subset \mathbb{R}^k$,

$$\lambda[B|\{T_n \in C\}] := \frac{\lambda[B \cap \{T_n \in C\}]}{\lambda[\{T_n \in C\}]};$$

a concave function μ on $\mathbb{R}^k$ (interpreted as the microcanonical entropy),

$$\mu(\boldsymbol{x}) := \inf_{G \ni \boldsymbol{x}} \liminf_n \frac{1}{|\Lambda_n|} \ln \lambda[\{T_n \in G\}],$$

the infimum being taken over a basis of neighbourhoods G of $\boldsymbol{x} \in \mathbb{R}^k$.

Theorem. *Let $C \subset \mathbb{R}^k$ be a closed convex subset with non–empty interior, such that there exists $\boldsymbol{x} \in C$ with $\mu(\boldsymbol{x}) > -\infty$. Then there exists $\boldsymbol{x}^* \in C$, such that $\mu(\boldsymbol{x}^*) = \sup_{\boldsymbol{x}\in C} \mu(\boldsymbol{x})$. Let $\boldsymbol{s}$ be a subgradient of the convex function $(-\mu)$ at $\boldsymbol{x}^*$. Then*

1) the sequence of averaged conditioned measures $\{\frac{1}{|\Lambda_n|}\sum_{j\in\Lambda_n} \theta_j \lambda[\,\cdot\,|\{T_n \in C\}]\}_n$ has at least one limit–point;

2) any limit–point of 1) is a $\boldsymbol{s}\cdot\boldsymbol{g}$– equilibrium state ($\boldsymbol{s}\cdot\boldsymbol{g} = \sum_{i=1}^k s_i g_i$).

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THE FALICOV-KIMBALL MODEL AS AN ISING MODEL

A. Messager [1]

The Falicov-Kimball model is a model of Quantum Statistical Mechanics which combines non moving ions and itinerating electrons, it is of interest in the solid state physics for the crystallisation and for the conductivity problems. The Hamiltonian is written in term of creations and annihilations operators of electrons and in term of a classical random variable defined at each site which is 1 if the site is occupied by an ion and o otherwise:

$$H_V = \sum_{x,y \in V} t_{xy} \left\{ C_x^\star C_y + C_y^\star C_x \right\} + 2U \sum_{x \in V} n_x W(x) + \mu^i \sum_{x \in V} (W(x) - \frac{1}{2}) + \mu^e \sum_{x \in V} (n_x - \frac{1}{2})$$

For large U and small μ^e we are able to show that the Falicov-Kimball model is equivalent to an effective Hamiltonian which is a long range antiferromagnetic Ising with a magnetic field in the following sense the free energies are the same and the correlation functions which are products of the variables $\sigma(x) = 2W(x) - 1$ are the same. The Hamiltonian can be computed exactly at the fourth order of a perturbation theory which derives from a cluster expansion in the Feynman-Kac representation (here the computation is done in space dimension 2):

$$\mathcal{H}_V^4 = (\mu_e - \mu_i) \sum_{x \subset V} \sigma_x + [\frac{1}{4U} - \frac{9}{32U^3}] \sum_{<x,y> \subset V} \sigma_x.\sigma_y + \frac{3}{16U^3} \sum_{<x,y> \subset V |x-y| = \sqrt{2}} \sigma_x.\sigma_y + \frac{1}{16U^3} \sum_{<x,y> \subset V, |x-y| = 2} \sigma_x.\sigma_y + \frac{5}{16U^3} \sum_{(x,y,z,t) \subset P(V)} \sigma_x.\sigma_y.\sigma_z.\sigma_t$$

The phase diagram of the Hamiltonian $\mathcal{H}_V^4$ is obtained from the Pirogov-Sinai theory: a set of phase transitions between phases with the same period occurs at low temperature. The phase diagram of the Falicov-Kimball (Pict.) is deduced from this of $\mathcal{H}_V^4$ with the following change: the tail of the effective potential corresponding to the higher order terms can be uniformly bounded by CU^{-5}, this creates gaps in the parameter $\mu^e - \mu^i$ where the phase diagram remain unknown. J think that in these gaps, by computing the Hamiltonian at the next order, one can prove the occurence of new phase transitions in domains which are again separated by new gaps (appearance of a "Devil staircase").

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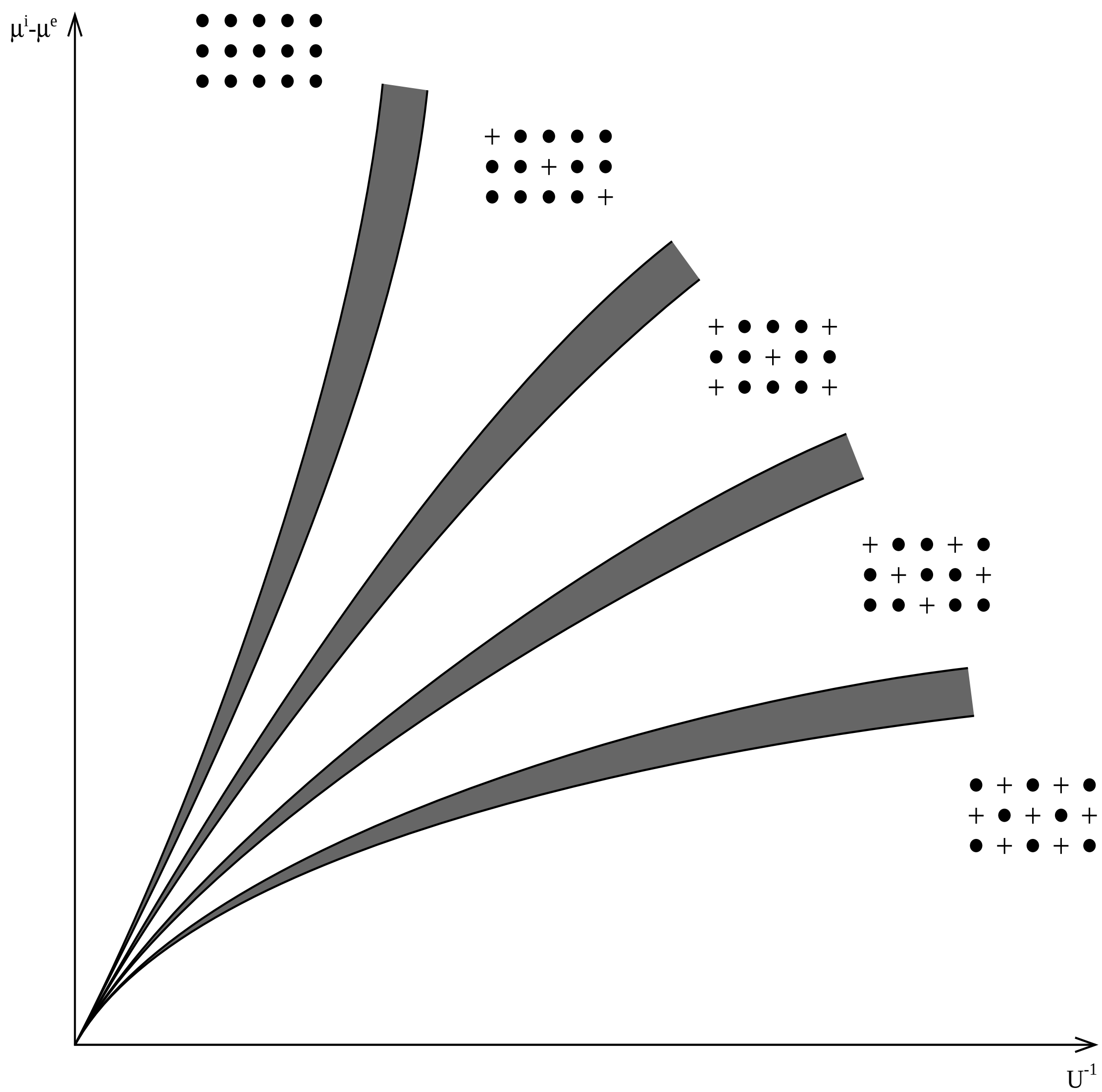

μ^i-μ^e
U^{-1}

ISING MODEL AND RENORMALIZATION GROUP PATHOLOGIES

E. Olivieri [1]

In this talk we report some recent results on the problem of well defining some renormalization maps for Ising-like systems.

In particular we discuss the Gibbsianness of the renormalized measures $\nu = T_b \mu_{\beta,h}$ arising from the application of the maps T_b ("on scale b") to the Gibbs measure $\mu_{\beta,h}$ for the Ising model at inverse temperature β and external magnetic field h.

1. When T_b is the decimation transformation on a sublattice of spacing b van Enter, Fernandez and Sokal (EFS) proved that ν is not Gibbsian for any given b for suitable large β and small h. In a joint paper with Fabio Martinelli we show that for any $h \neq 0$ and any sufficiently large β if b is chosen sufficiently large ν is Gibbsiann; moreover it tends to a trivial fixed point as $b \to \infty$.

2. When T_b is the block-average transformations (b is, in this case, the side of the blocks) (EFS) showed that for large β and any h, ν is non-Gibbsian. In another joint paper with Fabio Martinelli we introduce a " therapy" for this " pathology"; we show that by applying to ν a single block-decimation transformation we get a weakly coupled Gibbs measure and we are able to compute the renormalized potential via a convergent cluster expansion.

3. Cassandro and Gallavotti (CG)in an old paper proposed an approach to study the block-average transformation. (CG) showed (formally) that the renormalized potentials are computable as thermal averages of local quantities w.r.t an auxiliary Gibbs measure relative to an Ising system where, for any block B_i, there is the constraint that the corresponding magnetization m_i is equal to zero. (CG) raised the question of whether or not the constrained model is above its critical temperature. This was considered a necessary condition to give sense to their method. In a recent joint paper with Giosi Benfatto and Enzo Marinari we analyze numerically this problem; by traditional Montecarlo methods we show that indeed the critical temperature of the constrained model is lower than the one of the full Ising model. Moreover we try to use some finite size conditions (Dobrushin- Shlosman's uniqueness conditions) which, via some recent results by F. Martinelli, E. Olivieri and R. Schonmann, are alble to insure the possibility of rigorously pursue the (CG) program by computing the renormalizazed potentials as convergent series. We get preliminary promising results in this direction.

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A study of the metastable behavior of the stochastic Ising model in the joint limit in which h and T vanish

Roberto H. Schonmann [1]

In this talk I briefly reviewed some recent results concerning the relaxation pattern of stochastic Ising models in the vicinity of the phase transition region. In this region the evolution far from equilibrium is governed by the behavior of individual droplets and metastability effects arise due to the fact that only large droplets of the equilibrium phase tend to grow, but a free-energy barrier has to be overcome in order to form such supercritical droplets. While this picture is well accepted from an heuristic point of view and confirmed by computer simulations, there are substantial difficulties in proving the theorems that may be conjectured based on this ideas. The main part of the talk was concerned with rigorous results which refer to the asymptotic behavior in which both the temperature T and the external magnetic field h are small. In this regime one can show that the relaxation time grows as an exponential of $1/(Th^{(d-1)})$, where $d > 1$ is the space dimensionality. Moreover the exact rate of exponential grows is also obtained and is of the precise form predicted by the heuristic picture based on the behavior of individual droplets.

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Restricted variation problem and the Ising model

Senya Shlosman [1 2]

We consider the 2D Ising model under small positive magnetic field h in a square box with negative boundary conditions. We suppose that the temperature T is below the critical. In order to make the influence of the boundary conditions to be of the same order as that of the magnetic field, we have to consider the box of the size B/h with B some constant. Hence to perform the thermodynamic limit we have to take h to zero. We want to know the limiting behavior of our model.

Our results are the following: there exists the value $B_0(T)$ such that if $B < B_0(T)$, then the limiting state is the $(-)$-state. In the opposite regime, when $B > B_0(T)$, the central part of the box is occupied by the $(+)$-phase, while the corners of the box are occupied by the $(-)$-phase. The fraction of the box, occupied by the $(-)$-phase, is a function of B, which goes to zero as B increases. The contour, that separates the $(+)$-phase from the $(-)$-phase in this regime, has an asymptotic nonrandom shape, if scaled by a factor h/B.

The asymptotic shape in question is given by the solution of a certain variational problem. In isotropic case it is a problem of finding the shortest closed curve inside the unit square, which encircles the given area a. If $a > \pi/4$, the corresponding curve is different from the circle, and consists from four straight segments and four quartercircles (TV-screen). The Ising model problem corresponds to the anisotropic case, where the length has to be replaced by the integral of the surface tension function. The circle is replaced then by the Wulff shape.

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