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Some problems connected with interpolation and sampling of analytic functions.
(Joint work with Joaquin Ortega)

In a series of recent papers Seip, Seip- Wallstén, and Lyubarskii-Seip have characterized sets of interpolation and sampling for various classes of analytic functions in the complex plane and the unit disk. These theorems include the case of the classical Bergman spaces in the disk, and are particularily surprising since no precise characterization of the zero sets is known. We give different proofs and generalizations of parts of these theorems. The methods are based on Hörmander-type estimates for the $\bar{\partial}$-operator and generalizations of these estimates to uniform norms.

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX

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Transformations of Fourier-Laplace type and related holomorphy domains on the complex hyperboloid.

Let $X_n^{(c)}$ be the complexification of

$$X_n = \mathrm{SO}_0(1,n)/\mathrm{SO}_0(1,n-1).$$

The following notoins and results in $\mathbb{C}^n$ have their counterparts in $X_n^{(c)}$: tubes and holomorphy envelopes of related edge-of-the-wedge domains; theorems, which associate support properties with analyticity properties through an appropriate Fourier-Laplace type transformation. The results also include the relationship between the Laplace transforms of Volterra kernels on X_n and the Fourier-Legendre expansion of kernels on the sphere

$$S_n = \mathrm{SO}_0(n+1)/\mathrm{SO}_0(n)$$

with appropriate analytic continuation and growth properties in $X_n^{(c)} \times X_n^{(c)}$.

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Type functions for CR manifolds.

This is a common work with C. Denson Hill (Stony Brook). The purpose of this talk is to give an analytic disc approach to the notion of type of points for Cauchy-Riemann (CR) manifolds of arbitrary dimension and codimension. We define few versions of type function $\Phi(p, \delta)$ on CR manifolds $M \subset \mathbb{C}^n$, $p \in M$, $\delta > 0$. Very roughly speaking, the function Φ measures the maximum distance from the point p to the centers of analytic discs of radius at most δ with boundaries on M. We give geometric meaning of the functions, some properties, formulate consequences, applications, and give examples.

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A relative index for CR-structures.

Let M be a compact 3–manifold with an embeddable CR-structure, D_1. Let S_1 denote the Szegö projector onto $\ker D_1$. We show that a deformation of this structure, D_2 is embeddable if and only if the restriction

$$S_1 : \ker D_2 \longrightarrow \ker D_1$$

is a Fredholm operator. Let $\mathrm{Ind}(D_1, D_2)$ denote the index of this restriction. We show that this index is a well defined invariant of the geometric CR-structures, independent of the choices made in it's definition. This invariant is then applied to study the geometry of the space of embeddable CR-structures as a subspace of the all CR-structures.

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Complex symmetric spaces.

We consider K-invariant domains in "complex symmetric spaces", that is in Stein manifolds of the form $K^{\mathbb{C}}/L^{\mathbb{C}}$, where K and $L \subset K$ are compact Lie groups, forming a compact symmetric pair. The main results can be summarized in the following theorems.

Theorem 1. *Let $\Omega \subset\subset K^{\mathbb{C}}/L^{\mathbb{C}}$ be a K-invariant Stein domain in a complex symmetric space. Then*
(a) Ω contains a minimal orbit of type K/L;
(b) $Aut(\Omega)$ stabilizes a minimal K-orbit of type K/L;
(c) $Aut(\Omega)$ is a compact group.

Theorem 2.. *Let $\Omega_1,\ \Omega_2 \subset\subset K^{\mathbb{C}}/L^{\mathbb{C}}$ be K-invariant Stein domains in a complex symmetric space. Then Ω_1 and Ω_2 are biholomorphic if and only if there exists $F \in Aut(K^{\mathbb{C}}/L^{\mathbb{C}})$ such that $F(\Omega_1) = \Omega_2$.*

These theorems can be considered a natural generalization of certain results recently proved by P. Heinzner (On the automorphisms of special domains in $\mathbb{C}^n$, Indiana. Math. J., 41(3) (1992), 707-712) for bounded domains in $\mathbb{C}^n$ containing the origin and invariant under a linear action by a compact group, without invariant holomorphic functions.

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Hamiltonian mechanics and fundamental solutions for subelliptic operators.

I shall discuss a new formula for the fundamental solution of a second order subelliptic partial differential operator. This fundamental solution is given by an integral over the characteristic variety of an expression whose denominator is a Hamiltonian action function and whose numerator solves an associated second order transport equation. As an illustration I shall use this formula to derive the explicit fundamental solution for the sublaplacian associated to the hypersurface

$$\{\Im z_2 = |z_1|^{2k}\} \subset \mathbb{C}^2.$$

The formula is fully invariant and, in principle, applies to general subelliptic operators. In particular it inverts the $\overline{\partial_b}$-Laplacian and yields the Neumann kernel for the $\overline{\partial}$-Neumann problem. A similar formula yields the heat kernel for such operators explicitly, in particular its small time asymptotic, which is the highly singular classical action or the Carnot-Caratheodory metric. The formula yields explicit new path integrals and sheds light on some interesting but so far unsolved ordinary differential equations.
This is a joint project by Richard Beals, Bernard Gaveau and Peter C. Greiner.

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On the behavior of invariant metrics near boundary points where the Levi form has at most one degegnerate eigenvalue.

Assume that $\Omega = \{r < 0\}$ is a smooth bounded pseudoconvex domain in $\mathbb{C}^n$ and $q^0 \in \partial\Omega$ is a point of finite type , and the rank of the Levi form of $\partial\Omega$ is at least $n-2$. Then we give in a small neighborhood of q^0 precise estimates for the boundary behavior of the Bergman kernel function and the metrics of Bergman, Caratheodory and Kobayshi, which generalize those of Catlin in case of dimension two.

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Compactness of the Neumann operator for piece-wise smoothly bounded strictly pseudoconvex domains.

This is a report of a joint work with G. Henkin.
We prove the compactness of the Neumann operator for strictly pseudoconvex manifolds with piece-wise smooth boundary. This leads to the Hodge decomposition theorem and Fredholm property for Toeplitz operators.
The compactness of the Neumann operator was proved in the smooth case by J.J. Kohn, but if the boundary of a pseudoconvex domain contains an analytic disc, Catlin remarked that the Neumann operator is not compact.
The principal difficulty is that the density of smooth forms in the graph norm

$$\|f\|^2 + \|\overline{\partial} f\|^2 + \|\overline{\partial}^* f\|^2 \quad \text{in} \quad \text{Dom } \overline{\partial} \cap \text{Dom } \overline{\partial}^*,$$

which is always true for pseudoconvex domains with smooth boundaries, fails to be true in the case of piece-wise smooth boundaries.

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Splitting of the $\overline{\partial}$-complex.

Let $L(W)$ be a weighted space of locally square integrable functions, where the weight system W satisfies some technical assumptions. The splitting of the $\overline{\partial}$-complex with coefficients in $L(W)$ can then be characterized by means of a system of plurisubharmonic functions satisfying a specific uniform estimate. This condition can be evaluated in many concrete cases. This can be applied to show the existence of extension and interpolation operators for holomorphic functions on subvarieties, the complementation of ideals of holomorphic functions and the existence of continuous linear right inverses for partial differential operators and convolution operators.

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Algebraic approximations in analytic geometry.

In the talk we will approximate certain analytic objects (holomorphic mappings between affine algebraic varieties; Stein spaces with isolated singularities) by their algebraic counterparts.

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The Bergman and Szegö projections on convex domains.

We give estimates on the size of the Bergman and Szegö kernels, associated to a convex domain of finite type in $\mathbb{C}^n$, near the boundary diagonal. We also obtain estimates on arbitrary derivatives of the kernels. These estimates are essentially sharp and allow us to conclude that the projections are well behaved on many classical Banach spaces, e.g., the L^p Sobolev spaces. The main idea of the proof of these estimates is that subelliptic estimates on the $\overline{\partial}$-Neumann problem are invariant under a certain two parameter family of (local) biholomorphisms of the domain; this information is then transfered to the Bergman kernel via Hodge theory.

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Extension and lacunas of solutions of linear partial differential equations.

For a compact convex set K in $\mathbb{R}^n$ with $\overset{\circ}{K} \neq \emptyset$ let $\mathcal{E}(K)$ denote the Fréchet space of all C^∞-functions on K. We investigate when the analogue of Whitney's extension theorem holds for the zero solutions in $\mathcal{E}(K)$ of a given linear partial differential operator $P(D)$ with constant coefficients. For a compact convex set $Q \subset \mathbb{R}^n$ with $Q \supset K$ we show that for each $f \in \mathcal{E}(K)$ satisfying $P(D)f = 0$ there exists $F \in \mathcal{E}(Q)$ satisfying $P(D)F = 0$ and $F|_K = f$ if and only if the variety

$$V(P) := \{z \in \mathbb{C}^n \ : \ P(z) = 0\}$$

satisfies a condition of $PL(K, Q)$ Phragmén-Lindelöf type for the plurisubharmonic functions on $V(P)$. This is a different characterization than the one given by Kiselman in 1969. Moreover, its evaluation leads to various other characterizations if K in contained in $\overset{\circ}{Q}$.

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Partial differential equations for analytic functions on compact convex sets in $\mathbb{C}^N$.

Given a nonpluripolar compact convex subset K of $\mathbb{C}^N$, it is well known for more than thirty years that for each nonzero polynomial P and more general for each entire function P on $\mathbb{C}^N$ of order one and zero type, the partial differential operator $P(D) : A(K) \to A(K)$ is a surjective operator, which is continuous if the space $A(K)$ of all analytic functions on K is endowed with the natural inductive limit topology. We will investigate whether there is even a continuous linear right inverse for $P(D)$. This problem is connected with a certain extremal plurisubharmonic function which will be introduced for this purpose.

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The Banachspace $H^1(X, d, \mu)$ (isomorphic classification).

We present the isomorphic classification of atomic $H^1(X, d, \mu)$, where (X, d, μ) is a space of homogeneous type, hereby completing a line of investigation opened by the work of Bernard Maurey and continued by Lennart Carleson and Przemyslaw Wojtaszczyk.
The resulting isomorphic representatives are dyadic $H^1, (\Sigma H^1_n)_{l^1}$ and l^1; each isomorphic type is characterized by geometric properties of (X, d, μ).

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On the variation of the density and an application to interpolation problems.

Let $\Gamma \subset \mathbb{C}$ be any discrete subset satisfying $\inf\{\mathrm{dist}(z, \Gamma \setminus \{z\}) \mid z \in \Gamma\} > 0$ and let

$$D^+(\Gamma) = \overline{\lim_{r\to\infty}} \sup_{z\in\mathbb{C}} \frac{\#(\Gamma \cap \Delta(z,r))}{\pi r^2}.$$

Here $\Delta(z,r) = \{w \in \mathbb{C} \mid |z-w| < r\}$. Let $\lambda : \Delta \times \hat{\mathbb{C}} \longrightarrow \hat{\mathbb{C}}$ be any holomorphic family of C^1-diffeomorphisms of $\hat{\mathbb{C}}$ such that $\lambda(t, \infty) \equiv \infty$, and let $\Gamma_t = \{\lambda(z,t) | z \in \Gamma\}$.

Lemma. *If $D^+(\Gamma) > 0$, then $\log D^+(\Gamma_t)$ is continuous and subharmonic.*

Let $\tilde{\Gamma} = \{(t, \lambda(t,z)) \mid (t,z) \in \Delta \times \Gamma\}$.

Theorem. *If $D^+(\Gamma) > 0$, then $(\tilde{\Gamma}, e^{-cD^+(t)}\sqrt{-1}dt \wedge d\bar{t}|_{\tilde{\Gamma}})$ is*
*) *<u>a set of interpolation in the L^2 sense</u> for $(\Delta \times \mathbb{C}, e^{-cD^+(t)}\sqrt{-1}(dt \wedge d\bar{t} + dz \wedge d\bar{z}))$ for any $c > 1$, where $D^+(t) := D^+(\Gamma_t)$.*

*) <u>Definition.</u> Let M be a complex manifold, let dV_M be a volume form on M, and let $A^2(M, dV_M)$ be the space of square integrable holomorphic functions on (M, dV_M). A complex submanifold $S \subset M$ with a volume form dV_S is said to be a set of interpolation in the L^2 sense for (M, dV_M) if there exists a bounded linear map $I : A^2(S, dV_S) \longrightarrow A^2(M, dV_M)$ such that $I(f)|_S = f$ for all $f \in A^2(S, dV_S)$.

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Partial differential equations and analytic continuation.

Consider a linear partial differential equation "in the holomorphic category", that is the independent variables are complex, the functions appearing are holomorphic in some domain of $\mathbb{C}^n$, and the derivatives are complex derivatives.It is an important feature of such equations that a solution holomorphic in some domain is in general (by virtue of its satisfying the p.d.e.) holomorphically extendible to some larger domain, the exact description of which can sometimes be given explicitly from knowledge of the principal part of the differential operator and the domain of regularity of the coefficients. The first result of this kind, due to S. Kovalevskaya, says that a solution to the complexified "heat equation" holomorphic in a bidisk $D \times D'$ is automatically extendible to the "cylinder" $D \times \mathbb{C}$. This idea was cleverly adapted by Ivar Fredholm to proving the non-continuability of certain lacunary Taylor series (his analysis contained a flaw, which however can be corrected).In the present talk this idea, which has been dormant for 100 years, is adapted to a new context motivated by work of Szegö on the analytic continuation of Legendre series, and made possible by recent progress on partial differential operators in $\mathbb{C}^2$ whose principal part is the Laplace operator. The work described is being done jointly with Peter Ebenfelt and Dimitry Khavinson.

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Hypercomplex structures on the tangent bundles of hermitian symmetric spaces.

We study two complex structures I_* and J, defined on domains in the tangent bundle of a hermitian manifold. We define I_* using the complex structure on M, while J is constructed using the Riemannian metric on M. We show that if M is a hermitian symmetric space associated to a classical group then the pullback of J by a suitable diffeomorphism of domains in TM anticommutes with I_*. A corollary is the existence of a hypercomplex structure on a domain in TM.

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Toeplitz operators and geometric quantization in several complex variables.

Toeplitz operators in several variables are defined for symmetric domains, Siegel domains and the more general circular domains (with respect to a compact group action). We discuss the structure of the associated C^*-algebras generalizing well-known results of Gohberg, Berger, Coburn and others, and introduce the corresponding quantization procedure of Berezin type. As an application, one obtains an explicit description of the Berezin transform which generalizes the classical heat equation identity for euclidean domains.

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On Levi q-convexity.

It is known that for an open subset Ω of $\mathbb{C}^n$ with C^2 boundary $b\Omega$ if it is a domain of holomorphy then Ω enjoys some complex convexity, namely: if U is an open neighborhood of $b\Omega$ and $\rho \in C^2(U, \mathbb{R})$ is an arbitrary defining function (i.e. $U \cap \Omega = \{z \in U \mid \rho(z) < 0\}$ and $d\rho \neq 0$ on U) then the Levi form $L(\rho, z)$ is semipositive definite on the holomorphic tangent space $T_z(b\Omega)$ for all $z \in b\Omega$. The converse is also true (known as the Levi problem).
Existence of Stein neighborhood basis for $\overline{\Omega}$ was studied by Diederich and Fornæss (see [4] and the references therein). In particular they pointed out that $\overline{\Omega}$ does not always admit a Stein neighborhood basis and produced a beautiful example, the worm domain.
However recent work by Colţoiu and Diederich [2] showed that $\overline{\Omega}$ has a fundamental system of 2-complete neighborhoods (in case Ω is Levi convex).
The aim of this article is to naturally extend these results, replacing Levi convexity by Levi q-convexity and we shall prove the following results:

Theorem 1. *Let Ω be an open subset of $\mathbb{C}^n$ that is Levi q-convex. Then $\overline{\Omega}$ has a neighborhood basis of $(q+1)$-complete open sets.*

For $q = 1$ one gets a result from [2]. Furthermore the $(q+1)$-completeness obtained is the best possible since we have:

Theorem 2. *For any $q \geq 1$ there is a bounded domain $D \subset \mathbb{C}^{q+1}$ with C^∞ boundary that is Levi q-convex and its closure $\overline{D}$ does not have a neighborhood basis of q-complete open sets.*

We also mention the following generalization of Theorem 1:

Theorem 3. *Let M be a p-complete complex manifold and Ω an open subset that is Levi q-convex. Then*
(a) Ω is $(p+q+1)$-complete and
(b) $\overline{\Omega}$ has a neighborhood basis of $(p+q)$-complete open sets.

Part (a) extends a well-known result of Vigna Suria [7] where the case $p = 1$, i.e. M is a Stein manifold, according to our setup, is considered.
Also we note that in [1] statement (a) was proved if Ω has boundary of class C^3.

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Solution operators for linear partial differential operators of second order and fundamental solutions with support in a half space.

The lecture reports on joint work with R. Meise and B. A. Taylor. Let $P(x) \in \mathbb{C}[x_1, ..., x_n]$ be a polynomial of degree 2. The problem whether the partial differential operator $P(D)$ admits a continuous linear right inverse in $C^\infty(\mathbb{R}^n)$ is invariant under transformations of the form $A(x) = Sx + a$, where $S \in GL(n, \mathbb{R})$, $a \in \mathbb{C}$. By means of such a transformation and after multiplication with a constant, P may be written as $P(x) = Q(x_1, ..., x_r) + L(x_{r+1}, ...x_n) + C$, where $Q(x_1, ...x_r) = {x_1}^2 + ...$ is a nonsingular quadratic form , L linear and $Q \in \mathbb{C}$. Then we have:

Theorem. *$P(D)$ admits a continuous linear right inverse in $C^\infty(\mathbb{R}^n)$ if and only if Q and L are real and Q is indefinite or if $r = 1$ and $L = 0$.*

If Q is indefinite and L real, then for every characteristic half space H there exists a fundamental solution with support in $\bar{H}$ which can be explicitly given and the right inverse can be explicitly written down by means of these fundamental solutions. $P(D)$ has then also a right inverse in $C^\infty(H)$.

The general characterization given earlier by the same authors (Ann. Inst. Fourier, Grenoble **40**(1990), 619-655) is so, for equations of second order, transformed into an explicit classification with explicitly given solution operators.

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Interpolation bodies in $\mathbb{C}^n$.

Let Ω be a bounded domain in $\mathbb{C}^n$ and let $A(\Omega)$ denote the space of functions holomorphic on Ω and continuous on $\overline{\Omega}$. Put $\|f\| = \max_{\overline{\Omega}} |f|$, for $f \in A(\Omega)$. We fix an n-tuple of points $M_1, \ldots, M_n$ in $\overline{\Omega}$. The corresponding interpolation body is defined to be the set of all points $(w_1, \ldots, w_n)$ in $\mathbb{C}^n$ such that $\forall \epsilon > 0 \;\; \exists f \in A(\Omega)$ with $\|f\| \leq 1 + \epsilon$ and $f(M_j) = w_j, \; 1 \leq j \leq n$.
The case when Ω is the unit disk in $\mathbb{C}$ was treated by G. Pick in 1916 and interpreted in terms of operator theory by D. Sarason in 1967. We shall discuss results on interpolation bodies obtained in the last 5 years by the author jointly with B. Cole and K. Lewis, and related results.