

Trapped Surfaces in Cosmological Spacetimes

Edward Malec
Niall Ó Murchadha

Vienna, Preprint ESI 207 (1995)

March 17, 1995

Supported by Federal Ministry of Science and Research, Austria
Available via WWW.ESI.AC.AT

Trapped surfaces in cosmological spacetimes.

Edward Malec⁺* and Niall Ó Murchadha⁺

⁺*Erwin Schrödinger Institute, Vienna, Austria*

⁺*on leave from the Physics Department, University College, Cork, Ireland*

^{*}*on leave from Institute of Physics, Jagellonian University, 30-059 Cracow, Reymonta 4, Poland*

Abstract

We investigate the formation of trapped surfaces in cosmological spacetimes, using constant mean curvature slicing. Quantitative criteria for the formation of trapped surfaces demonstrate that cosmological regions enclosed by trapped surfaces may have matter density exceeding significantly the background matter density of the flat and homogeneous cosmological model.

04.20.Me, 95.30.Sf, 97.60.Lf, 98.80.Dr

Typeset using REVTeX

I. INTRODUCTION

In our previous work ([1]) we investigated the formation of trapped surfaces in various cosmological models. Recently we have found a particularly useful formulation ([2]) of the spherically symmetric Einstein constraint equations that allowed us to improve our early estimates ([3]) for conditions determining the appearance of trapped surfaces. In the present paper we apply the new formalism to spherically symmetric cosmologies. As a result we find stronger criteria in spacetimes that were investigated previously and, more importantly, we are able to deal with hyperbolic universes where our previous attempts have failed.

The order of the article is as follows. The first section presents the formalism. In Section 2 we deal with the main results. Section 3 shows that regions enclosed by trapped surfaces must be invisible to external observers. The last Section contains conclusions of which the most important is that energy density inside cosmological regions enclosed by trapped surfaces may exceed significantly the average energy density of the flat and homogeneous cosmological model.

There exist three homogeneous spherically symmetric cosmologies. The three are

i) the closed (k=1) cosmology with metric

$$ds^2 = -d\tau^2 + a^2(\tau)[dr^2 + \sin^2 r d\Omega^2], \quad (1)$$

ii) the open flat (k=0) cosmology with metric

$$ds^2 = -d\tau^2 + a^2(\tau)[dr^2 + r^2 d\Omega^2], \quad (2)$$

iii)

$$ds^2 = -d\tau^2 + a^2(\tau)[dr^2 + \sinh^2 r d\Omega^2], \quad (3)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ is the standard line element on the unit sphere with the angle variables $0 \leq \phi < 2\pi$ and $0 \leq \theta \leq \pi$.

The geometric part of the initial data set of the Einstein equations consists of the intrinsic three-geometry and the extrinsic curvature K_{ab} which is essentially the first time derivative of the metric, all given at some time (say $\tau = 0$). The intrinsic geometries are respectively

$$a^2(\tau)[dr^2 + \sin^2 r d\Omega^2], \quad (4)$$

$$a^2(\tau)[dr^2 + r^2 d\Omega^2], \quad (5)$$

$$a^2(\tau)[dr^2 + \sinh^2 r d\Omega^2], \quad (6)$$

and in each case the extrinsic curvature is pure trace

$$K_{ab} = H g_{ab} \quad (7)$$

where H is a time dependent function that is constant on each slice $\tau = \text{const}$. It is called the Hubble constant and it is given by $H = \frac{\partial_\tau a}{a}$.

In the general case initial data consist of the quartet $(g_{ij}, K_{ij}, \rho, J_i)$ where g_{ij} is the intrinsic metric, K_{ij} is the extrinsic curvature, ρ is the matter energy density and J_i is the matter current density. These cannot be given arbitrarily but must satisfy the constraints

$${}^{(3)}\mathcal{R} - K_{ij}K^{ij} + (tr K)^2 = 16\pi\rho \quad (8)$$

$$\nabla_i K^{ij} - \nabla^j tr K = -8\pi J^j \quad (9)$$

where ${}^{(3)}\mathcal{R}$ is the scalar curvature of the intrinsic metric.

The momentum constraint, (9), is identically satisfied in the case of homogeneous cosmologies (with $J_i = 0$) and the hamiltonian constraint, (8), reduces to

$$16\pi\rho = \frac{6k}{a^2} + 6H^2 \quad (10)$$

where k is 1, 0, -1 in the closed, flat and hyperbolic cosmologies, respectively. Thus we can conclude that all slices of the constant coordinate time have a uniform energy density ρ_0 which is at rest.

In this article we wish to consider data for spherically symmetric cosmologies which either in the large approximate the standard cosmologies or asymptotically approach them. In all cases we will make the assumption that the initial slice is chosen so that the trace of the extrinsic curvature is constant on the slice. In order to retain the link with homogeneous cosmologies we define $trK = 3H$.

The initial data we consider is a spherically symmetric set consisting of a three-metric

$$ds_{(3)}^2 = a^2 dr^2 + b^2(r) f^2(r) d\Omega^2, \quad (11)$$

an extrinsic curvature

$$K_r^r = H + K(r), \quad K_\theta^\theta = H - K/2, \quad K_\phi^\phi = H - K/2 \quad (12)$$

an energy density $\rho(r)$ and a current density j_i . The function $f(r)$ will be one of the set $\sin(r), r, \sinh(r)$, depending on the type of cosmology.

There are some useful geometric quantities that can be defined. One of them is the proper distance from the center of symmetry given by $dl = adr$. The Schwarzschild (areal) radius R is given by $R = bf$. The mean curvature of a centered two-sphere as embedded in an initial three dimensional hypersurface is

$$p = \frac{2\partial_t R}{R}. \quad (13)$$

In a general spacetime we may investigate the geometry by considering the propagation of various beams of light rays through a space-time. These beams in general will shear and either expand or contract; a number of (optical) functions will be required to describe their propagation. In a spherically symmetric spacetime we focus our attention to light rays moving orthogonally to two-spheres centered around a center of symmetry. We need only two functions. These are the divergence of future directed light rays

$$\theta = \frac{2}{R} \Big|_{out} \frac{d}{d\tau_{out}} R \quad (14)$$

and the divergence of past directed light rays

$$\theta' = \frac{-2}{R} \frac{d}{d\tau_{in}} R \quad (15)$$

where $\frac{d}{d\tau_{out}}$ is the derivative along future-pointing outgoing radial null rays and $\frac{d}{d\tau_{in}}$ is the derivative along future-pointing ingoing radial null rays. One interesting property of θ and θ' is that they can be expressed purely in terms of initial data on a spacelike slice. In the spherically symmetric case we have

$$\theta = p - K_r^r + trK = p - K + 2H, \quad (16)$$

and

$$\theta' = p + K_r^r - trK = p + K - 2H. \quad (17)$$

This means that θ and θ' are three-dimensional scalars. They are not four-scalars, since they depend on a choice of affine parameters along the null rays.

In the homogeneous universes we find that $pR = 2$, $pR = 2 \cos(r)$ and $pR = 2 \cosh(r)$ in the $k = 0, 1, -1$ cases respectively and

$$R\theta = 2 + 2RH = 2 + 2\sqrt{\frac{8\pi\rho_0}{3}}R, \quad R\theta' = 2 - 2RH \quad (18)$$

for $k=0$,

$$\begin{aligned} R\theta &= 2 \cos(r) + 2RH = 2 \cos(r) + 2aH \sin(r) = 2 \cos(r) + 2\sqrt{\left(\frac{8\pi\rho_0 a^2}{3} - 1\right)} \sin(r), \\ R\theta' &= 2 \cos(r) - 2\sqrt{\left(\frac{8\pi\rho_0 a^2}{3} - 1\right)} \sin(r) \end{aligned} \quad (19)$$

for $k=1$,

$$\begin{aligned} R\theta &= 2 \cosh(r) + 2RH = 2 \cosh(r) + 2\sqrt{\left(\frac{8\pi\rho_0 a^2}{3} + 1\right)} \sinh(r), \\ R\theta' &= 2 \cosh(r) - 2\sqrt{\left(\frac{8\pi\rho_0 a^2}{3} + 1\right)} \sinh(r) \end{aligned} \quad (20)$$

for $k=-1$.

A surface on which θ is negative is called, after Penrose [4]), a future trapped surface and a surface on which θ' is negative is called a past trapped surface. The occurrence of

such surfaces in a spacetime is an indication of the fact that the gravitational collapse is well advanced. In the case of homogeneous closed cosmologies future trapped surfaces exist for any $r > \cot^{-1}(aH)$. In neither $k=0$ nor $k=-1$ is $R\theta$ ever negative if $H > 0$.

In this article we consider a universe that is homogeneous in the large but that it is dotted with numerous spherical inhomogeneities, far from each the metric approaches the background metric of a homogeneous universe. If we center our coordinate system at a particular lump we expect that optical scalars approach the values given in (18, 19, 20) far away from the lump. In the case of closed cosmologies this limiting value is expected to be met for values of the coordinate radius r much less than $\pi/2$.

We assume local flatness at the origin, i. e., $\lim_{R \rightarrow 0} R\theta = \lim_{R \rightarrow 0} R\theta' = 2$ although this condition can be relaxed to allow for a conical singularity there, i. e., $0 < \lim_{R \rightarrow 0} R\theta, \lim_{R \rightarrow 0} R\theta' \leq 2$.

II. MAIN CALCULATIONS.

The spherical initial data must satisfy the constraints, which read, in terms of functions θ and θ'

$$\partial_l(\theta R) = -8\pi R(\rho - j) - \frac{1}{4R}[2(\theta R)^2 - \theta R\theta' R - 4 - 12\theta RHR] \quad (21)$$

$$\partial_l(\theta' R) = -8\pi R(\rho + j) - \frac{1}{4R}[2(\theta' R)^2 - \theta R\theta' R - 4 + 12\theta RHR] \quad (22)$$

where $j = j_l$ is the radial component of the matter current density normalized so that $j^2 = j^k j_k$. We can manipulate equations (21) and (22) to obtain

$$\partial_l(\theta' R\theta R) = -8\pi(\rho(\theta' R + \theta R) + j(\theta R - \theta' R)) - \frac{1}{2R}[(\theta R\theta' R - 4)(\theta' R + \theta R)]. \quad (23)$$

Let us now assume that the total matter satisfy the dominant energy condition, i. e., $\rho \geq |j|$. Assume that $\theta R\theta' R > 4$ at a particular point. Consider first the situation where both θR and $\theta' R$ are positive. Then $(\theta' R + \theta R) > (-\theta' R + \theta R)$ and $\rho(\theta' R + \theta R) + j(\theta' R + \theta R) \geq 0$.

This means that both terms of (23) are nonpositive and the derivative of the product $\theta R\theta'R$ is negative. On the other hand, when both θR and $\theta'R$ are negative and their product is greater than 4, then $\rho(\theta'R + \theta R) + j(\theta'R + \theta R) < 0$ and the first term in (23) is positive. The second term becomes also positive, so that $\partial_i(\theta R\theta'R) > 0$. Thus in both cases if $\theta R\theta'R > 4$ then $\partial_i(\theta R\theta'R) \neq 0$.

Let us now consider the expressions for the product of the two scalars $\theta R\theta'R$ in each of the three homogeneous cosmologies. We get

$$R\theta R\theta' = 4 - 4R^2 H^2 \quad (24)$$

for $k=0$,

$$R\theta R\theta' = 4 \cos^2(r) - 4R^2 H^2 \quad (25)$$

for $k=1$ and

$$R\theta R\theta' = 4 \cosh^2(r) - 4\left(\frac{8\pi\rho_0 a^2}{3} + 1\right) \sinh^2(r) = 4\left[1 - \frac{8\pi\rho_0 a^2}{3}\right] \sinh^2(r) \quad (26)$$

for $k=-1$.

In each of these cases we have $R\theta R\theta' = 4$ at the origin and never more than 4. We are considering initial geometries that locally are flat and asymptotically approach the homogeneous cosmologies, so that both at the origin and far from the center the product $R\theta R\theta'$ does not exceed 4. If it were to achieve a maximal value greater than 4 somewhere in between, then its derivative would have to vanish; but that is excluded in the preceding analysis. Therefore we have proven:

Lemma 1. Assume that matter satisfies the dominant energy condition and that spherical cosmological data are locally flat at the center and are asymptotic to any of standard homogeneous cosmological models. Then

$$R\theta R\theta' \leq 4.$$

Remarks:

i) The above statement is true for any regular slice, with arbitrary (i. e., nonconstant on a part of a slice) $\text{tr}K$, assuming that the slice is asymptotic to a homogeneous constant mean curvature slice.

ii) It implies the positivity of the Hawking mass on a sphere centered around a symmetry center; $2M_H = R(1 - \frac{R\theta R\theta'}{4})$ cannot become negative on a fixed slice.

Lemma 1 holds true for all three cosmological models.

The main issue that we will address in this paper is the question of the formation of trapped surfaces due to concentration of matter. The result will be obtained through a careful analysis of (21). What we do is multiply (21) by R , use (13) and write the resulting equation in the following form

$$\partial_l(\theta R^2) = -8\pi R^2(\rho - j) + 1 + \frac{1}{2}\theta R\theta' R - \frac{1}{4}(\theta R)^2 + 3\theta RHR. \quad (27)$$

The substitution of (16) and (17) into (27) gives

$$\partial_l(\theta R^2) = -8\pi R^2(\rho - \frac{3H^2}{8\pi} - j) + 1 + \frac{1}{4}(pR + KR)^2 - R^2K^2 + 2R^2Hp \quad (28)$$

or

$$\partial_l(\theta R^2) = -8\pi R^2(\rho - \rho_0 + \frac{3k}{8\pi a^2} - j) + 2 - (1 - \frac{1}{4}(pR + KR)^2) - R^2K^2 + 4RH\partial_l R, \quad (29)$$

where we used the relation (10) to eliminate the H^2 term and use the definition of mean curvature p .

Let us integrate (29) from the origin out to a surface S . We identify

$$\Delta M = 4\pi \int_0^{L(S)} R^2(\rho - \rho_0)dl = \int_{V(S)} dV(\rho - \rho_0) \quad (30)$$

as the excess matter inside a volume $V(S)$ bounded by S and

$$P = 4\pi \int_0^{L(S)} R^2 j dl = \int_{V(S)} dV j \quad (31)$$

as the total radial momentum inside S . In this notation, the aforementioned integration yields

$$\theta R^2|_S = -2(\Delta M - P) - \frac{3k}{4\pi a^2}V + 2L + \frac{HA}{2\pi} - \int_{V(S)} dV \left(1 - \frac{1}{4}(pR + KR)^2 + R^2 K^2\right) \quad (32)$$

where A is the area of the surface S and L is the geodesic distance of S from the centre. Below we will prove, in a series of lemmas, that under some conditions we can control the sign of the last integral.

Lemma 2. Assume $k=0, 1$ cosmologies which are locally flat. If the energy condition $\rho - \rho_0 - |j| \geq -\frac{3k}{4\pi a^2}$ is satisfied out to an asymptotic region then

$$2 \geq |pR + KR|, \quad 2 \geq |pR - KR|.$$

Lemma 3. In a data set that approaches the $k = -1$ locally flat cosmology, if the energy condition $\rho - \rho_0 - |j| \geq \frac{3}{4\pi a^2}$ is satisfied inside a sphere S then

$$2 > (pR + KR), \quad 2 > (pR - KR).$$

Before proving the two lemmas, let us formulate two main results that give sufficient conditions for the formation of trapped surfaces.

Theorem 1. Given data which approaches either the $k=0$ or the $k=1$ locally flat cosmology, if the energy condition $\rho - \rho_0 - |j| \geq \frac{3k}{4\pi a^2}$ is satisfied out to an asymptotic region and if

$$\Delta M - P \geq -\frac{3k}{8\pi a^2}V + L + \frac{HA}{4\pi} \quad (33)$$

at a surface S then S is future trapped.

Proof of theorem 1: the result follows directly from eq. (32) and the estimate of Lemma 2.

Theorem 2. Assume that normally ingoing light rays are everywhere convergent inside a volume V bounded by a surface S , $\theta' > 0$. Given data which approaches the $k = -1$ locally flat cosmology, if the energy condition $\rho - \rho_0 - |j| \geq \frac{3}{4\pi a^2}$ is satisfied inside the volume V and if

$$\Delta M - P \geq \frac{3}{8\pi a^2} V + L + \frac{HA}{4\pi} \quad (34)$$

at the surface S then there exists a surface inside S that is future trapped.

Proof of Theorem 2. Assume that there is no future trapped surface inside S , i. e., $\theta = p + 2H - K > 0$. Since we also assume that there is no past trapped surface, we may conclude that inside S $p - K > -2H, p + K > 2H$. We know that p is positive inside S because we have that $p = (\theta + \theta')/2$ and each of θ and θ' is positive. We also have $2 > pR - KR > -2HR$ and $2 > pR + KR > 2HR$; the last inequalities follow from lemma 3. If $H > 0$ we have that $pR + KR$ is positive and thus $(p + K)^2 R^2 \leq 4$ and the last integral of (32) is strictly negative. On the other hand, if $H < 0$ we must have that $pR - KR$ is positive and $(pR - KR)^2 \leq 4$ but we could have that $pR + KR$ be negative. This can only happen while K is negative since we know that p is positive. In this case we write the integrand of (32) as $1 - \frac{1}{4}(pR - KR)^2 - pKR^2 + K^2R^2$. This is clearly nonnegative. Thus we also have in this case that the last term in (32) is negative. This contradicts the assumption that there is no trapped surface. Hence, under the assumptions of Theorem 2, there must exist a trapped surface inside S .

In order to prove lemmas 2 and 3 we shall return to equations (21) and (22) and write them in terms of Rp, RK and RH . (21) can be written as

$$\partial_t(pR - KR) = -8\pi R(\rho - j - \frac{3H^2}{8\pi}) - \frac{1}{2R}(Rp - RK)^2 + \frac{1}{4R}(Rp - RK)(Rp + RK) + \frac{1}{R} \quad (35)$$

and (22) as

$$\partial_t(pR + KR) = -8\pi R(\rho + j - \frac{3H^2}{8\pi}) - \frac{1}{2R}(Rp + RK)^2 + \frac{1}{4R}(Rp - RK)(Rp + RK) + \frac{1}{R}. \quad (36)$$

We will prove first the upper bound of $pR + KR, pR - KR$, simultaneously for both Lemma 2 and 3; this part of the proof does not depend on the type of a cosmological spacetime. Also, as it will become clear, the energy condition shall be imposed only inside a sphere S if

we are interested in finding the estimate inside S (as opposed to the estimations from below that require the global assumption made in Lemma 2). According to the conditions made in lemmas, the first term of either equation (35) or (36) is nonpositive. We show that in the situation of interest the remainders of each of the equations are also nonpositive.

At the center of symmetry the quantities $pR+KR$, $pR-KR$ are equal to 2, for all types of cosmology. This means that right hand sides of either (35) or (36) must be nonpositive and that the quantities in question start from the origin with the value 2 and start to decrease as soon as they meet either positive $\rho + j - \frac{3H^2}{8\pi}$ or $\rho - j - \frac{3H^2}{8\pi}$.

Let us assume that further out one of the two, say $pR+KR$, rises up to 2 with $pR-KR$ lagging behind. In this case we can write the non-material part of the right hand side of (36) as follows

$$-\frac{1}{2}(Rp + RK)^2 + \frac{1}{4}(Rp - RK)(Rp + RK) + 1 = -1 + \frac{1}{2}(Rp - RK) \leq 0. \quad (37)$$

Because the material part of (36) is nonpositive, we get that $\partial_t(Rp + RK) \leq 0$ so that $pR + KR$ cannot exceed 2. A similar argument can be made for $pR - KR$. Thus Lemma 3 and the upper bound of Lemma 2 are proven; as is clear from the above derivation, in order to have a bound that is valid inside a sphere S we need the energy condition that is imposed only inside S .

The same reasoning can be applied to complete the proof of Lemma 2. We will show, that if one of the two quantities in question reaches the value -2, then at least one of them must be less than -2, thus breaking either the demand of geometries being asymptotic to a homogeneous cosmology in the sense expressed in equations (18) and (19).

In order to show this we need the global energy condition of Lemma 2. Let us assume that there exists a point where $pR + KR = -2$, with $pR - KR \geq pR + KR$. Then the nonmaterial part of Eq. (36) reads

$$-\frac{1}{2}[2(Rp + RK)^2 + \frac{1}{4}(Rp - RK)(Rp + RK) + 1] \leq 0 \quad (38)$$

Eq. (36) implies now (assuming the energy condition) that $pR + KR$ has to become more negative, if $pR + KR < pR - KR$ and may stay at -2 in the case of equality only if the matter

contribution exactly cancels. However, if we can impose an outer boundary condition such that $pR + KR \geq -2$ then we get a contradiction. A similar argument works for $pR - KR$. The outer boundary condition is guaranteed in the cases of interest. Cosmological spacetime dotted with inhomogeneities have the property that asymptotically $pR + KR$ and $pR - KR$ approach values given by (18) and (19) which must be strictly bigger than -2. That ends the proof of Lemma 2.

It is interesting that we obtain an exact criterion with the constant 1; this suggests that the above theorem constitute a part of a more complex true statement that can be formulated for general nonspherical spacetimes. It suggests also that $M(S)$ is a sensible measure of the energy of a gravitational system that might appear as a part of a quasilocal energy measure in nonspherical systems.

It is clear that the analysis performed here can include cases where the sources are distributions rather than classical functions; in particular, we have no difficulty with shells of matter. All we get on crossing the shell is a downward step in θ and θ' . More interestingly, we can extend the analysis to include conical singularities at the origin ([6]), in a way analogous to that described in ([2]).

III. CONFINING PROPERTY OF TRAPPED SURFACES.

In this Section we show that a region enclosed by trapped surfaces cannot be seen by external observers. This fact has been proven (without referring to the Cosmic Censor Hypothesis) by Israel ([5]). Here we will present a different version of the proof that is based on a 1+3 decomposition of a spacetime (as opposed to the proof of Israel, who used a 2+2 decomposition).

We need the evolution part of the Einstein equations and the lapse equation. These are

$$\partial_t(\delta K_r^r - 2H(t)) = \frac{3\alpha}{4}(\delta K_r^r)^2 - \frac{\alpha p^2}{4} - \frac{p}{\sqrt{a}}\partial_r\alpha + \frac{\alpha}{R^2} + 8\pi\alpha T_r^r + 3\alpha H^2 - 3H\delta K_r^r \quad (39)$$

and

$$\nabla_i \partial^i \alpha = \alpha \left(\frac{3}{2} (\delta K_r^r)^2 4\pi (\rho + T_i^i) + 3H^2 \right) + 3\partial_t H. \quad (40)$$

In addition we need the evolution equation of the mean curvature p of centred spheres

$$\partial_t p = \frac{\partial_r \alpha}{\sqrt{a}} (-\delta K_r^r + 2H) + 8\pi \alpha \frac{\dot{j}_r}{\sqrt{a}} + \frac{p\alpha}{2} (\delta K_r^r - 2H) \quad (41)$$

Using these equations we can find the full time derivative of θ along a trajectory of null geodesics normal to centred spheres

$$\left(\partial_t + \frac{\alpha}{\sqrt{a}} \right) \theta = \frac{\partial_r \alpha}{\sqrt{a}} \theta' \theta - 8\pi \alpha \left(-2 \frac{\dot{j}_r}{\sqrt{a}} + \rho + T_r^r \right) - \alpha \theta^2 + 3\alpha H \theta. \quad (42)$$

Take now an apparent horizon, i. e., a centred sphere S of vanishing $\theta(S)$; (42) implies that photons that start from S will forever remain inside an apparent horizon, if the strong energy condition $-2 \frac{\dot{j}_r}{\sqrt{a}} + \rho + T_r^r \geq 0$ is assumed. Hence apparent horizons move faster than light in cosmological spacetimes (in contrast with asymptotically flat spacetimes, where they can eventually stabilize to the speed of light); they act as one-way membranes for non-tachyonic matter. This means that outside observers cannot detect any information from any inside region that is enclosed by a trapped surface. The only way to draw any conclusions about a piece of a spacetime that is enclosed by a trapped surface is through the observation of "long-wave" effects - through the attractive force that large massive objects exert on their surrounding.

IV. DISCUSSION.

Cosmological trapped surfaces that we discuss in preceding sections can, if they exist, accumulate an enormous amount of energy. Typically, as we have shown, the matter content of a trapped surface having a geodesic radius L is of the order L plus the background energy $M_H = \frac{3H^2 V}{8\pi}$ (we neglect here the effects related to the possibility of a nonzero curvature of the space-like slice and the surface term $\frac{HA}{2\pi}$). Assume that there exists a trapped surface with a proper radius of the order of 1000 megaparsecs. Then its excess energy is of the order of 1000 (in units of megaparsecs). The present value of the Hubble

constant is about $50 \frac{km}{s * megaparsec}$ or (in units in which the speed of light $c=1$) $\frac{1}{6000 megaparsec}$. Therefore the expected value of the background energy inside the above ball is of the order $0.5 * (\frac{1}{6000})^2 megaparsec = 14 magaparsec$, which is about 10^2 times less than the energy content that is needed in order to form, say, a spherical massive shell that creates a trapped surface. We include this crude calculation just to point out that the formalism of general relativity does allow for cosmological regions with high concentrations of matter that are in principle invisible by external observers.

ACKNOWLEDGMENTS

This work was partially supported by Forbairt grant SC/94/225 and the KBN grant 2 PO3B 090 08.

REFERENCES

- [1] U. Brauer and E. Malec, Phys. Rev. **D45**, R1836(1992); E. Malec and N. Ó Murchadha, Phys. Rev. **D47**, 1454(1993); E. Malec, U. Brauer and N. Ó Murchadha, Phys. Rev. **D49**, xxxx(1994).
- [2] E. Malec and N. Ó Murchadha, Phys. Rev. **D50**, R6033(1994).
- [3] P. Bizoń, E. Malec and N. Ó Murchadha, Phys. Rev. Lett. **61** 1147(1988); Class. Quantum Grav. **6**, 961 (1989); **7**, 1953(1990).
- [4] R. Penrose, Phys. Rev. Lett. **14**, 57 (1965).
- [5] W. Israel, Phys. Rev. Lett. **56**, 86(1986).
- [6] J. Guven and N. Ó Murchadha, to be published.