

# Choquet integrals, stochastic dominance with respect to a capacity and risk measures

Miryana Grigorova

Laboratoire de Probabilités et Modèles Aléatoires  
Université Paris-Diderot (Paris 7)

In the framework of ambiguity, the notion of capacity (or non-additive probability) replaces that of probability measure, and Choquet integrals replace the usual mathematical expectations. In this talk, we will recall the notion of capacity (which is a monotone, normalized set function), and that of Choquet integral with respect to a capacity (which could be seen as a generalization of the Lebesgue integral). Then, we will extend the notions of increasing, and increasing convex stochastic dominance, well-known in the case of a probability measure, to the more general setting of a capacity. We will characterize these "generalized" relations in terms of distribution functions and quantile functions with respect to the initial capacity. We will then consider the classes of monetary risk measures (defined on the space of bounded measurable functions) having the properties of comonotonic additivity and consistency with respect to a given "generalized" stochastic dominance relation. These classes of risk measures will be characterized in terms of Choquet integrals with respect to a "distortion" of the initial capacity. A "Kusuoka-type" characterizations of the class of monetary risk measures having the properties of comonotonic additivity and consistency with respect to the "generalized" increasing convex stochastic dominance will also be established. A generalization of Hardy-Littlewood's inequalities to the case of a capacity will be used as a tool.

The talk will be based on the following two documents :

M. Grigorova, "Stochastic orderings with respect to a capacity and an application to a financial optimization problem", available at <http://hal.archives-ouvertes.fr/hal-00614716>.

M. Grigorova, "Stochastic dominance with respect to a capacity and risk measures", available at <http://hal.archives-ouvertes.fr/hal-00639667>.