Universität Wien SS 2013
Fakultät für Mathematik
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Mathematical Finance 2

Exercise sheet 4

1. Let (W_t) denote a standard Brownian motion and $I = [a, b]$ a compact interval. Show that

$$\mathbb{P}\left[\frac{W_{t+h} - W_t}{h} \in I\right] \to 0 \quad \text{as } h \to 0.$$

What does this precisely mean for the differentiability of the Brownian paths?

2. Solve Exercise 3.4 of Shreve's book.

3. Let W_t be standard Brownian motion. Show, using Itô's formula, that the following processes are martingales.

a) $X_t = \mathrm{e}^{t/2} \cos W_t$

b) $X_t = \mathrm{e}^{t/2} \sin W_t$

c) $X_t = (W_t + t) \exp(-W_t - \frac{1}{2}t)$

Website: http://www.mat.univie.ac.at/∼finance_hp/exercisesSS13_MF.html