Universität Wien SS 2013
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Mathematical Finance 2
Exercise sheet 6

1. Let $B_1(t)$ und $B_2(t)$ be correlated Brownian motions with

$$dB_1(t)dB_2(t) = \varrho(t)dt,$$

where $\varrho(t)$ is a process with values in $]-1, 1[$. Define processes W_1 und W_2 so that

$$\begin{aligned} B_1(t) &= W_1(t), \\ B_2(t) &= \int_0^t \varrho(s)dW_1(s) + \int_0^t \sqrt{1-\varrho^2(s)}dW_2(s). \end{aligned}$$

Show that W_1 and W_2 are independent Brownian motions.

2. Solve Exercise 4.15 in Shreve's book.

3. (A discrete analog of Lévy's characterization of Brownian motion.) Let $(M_n)_{n=0}^{\infty}$ be a discrete-time martingale with quadratic variation

$$\sum_{k=0}^{n-1}(M_{k+1} - M_k)^2 = n.$$

Suppose further that $M_n = \sum_{k=0}^{n} X_k$, where (X_k) is a sequence of independent random variables. Show that M is a Bernoulli process.

Website : http ://www.mat.univie.ac.at/∼finance_hp/exercisesSS13_MF.html