

## Mathematical Finance 2

### Exercise sheet 7

1. Solve Exercise 4.18 of Shreve's book.
2. Solve Exercise 4.19 of Shreve's book.
3. Let  $W_1, \dots, W_n$  be independent standard Brownian motions, and define the process  $X$  by  $X(t) = h(W_1(t), \dots, W_n(t))$  for some bounded function  $h : \mathbb{R}^n \rightarrow \mathbb{R}$ . Show that  $X$  is a martingale (submartingale) if  $h$  is harmonic (subharmonic).

*Hint* : A function  $h(x_1, \dots, x_n)$  is said to be *harmonic* resp. *subharmonic* if it satisfies the condition

$$\sum_{i=1}^n \frac{\partial^2 h}{\partial x_i^2} = 0 \quad \text{resp.} \quad \geq 0.$$

**Website** : [http://www.mat.univie.ac.at/~finance\\_hp/exercisesSS13\\_MF.html](http://www.mat.univie.ac.at/~finance_hp/exercisesSS13_MF.html)