Gromov's a-T-menability and free-by-free groups

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Abstract: While trying to prove the "metric-approximation property" (a C^*-algebra property) for free groups, at the end of the seventies Haagerup proved that any finitely generated free group admits a proper conditionnally negative definite function. This property appeared to have some interest on its own and was then called "Haagerup property". It was later rediscovered by Gromov as a kind of converse to the more famous Kazhdan property (T) (a group having both Haagerup and Kazhdan properties is a compact group) and was then termed Gromov's a-T-menability. Amenable groups are a-T-menable. Since then a-T-menability has been intensively studied, however not so many thing are known about its preservation under extension: there are examples of abelian-by-(free non abelian groups) which are not Haagerup (Burger, de la Harpe-Valette) whereas any (a-T-menable)-by-amenable group is (Jolissaint). A natural question is whether the semi-direct product of two free non abelian groups is a-T-menable. The aim of this talk is to give a first non-trivial example of such an a-T-menable group. The main ingredient of the proof is the construction of a space with walls structure as introduced by Haglund-Paulin. If time permits we will try to show why we can hope to prove that any such extension over polynomially growing automorphisms of the free group is a-T-menable.