Dynamics, growth of cocycles, and distorted elements

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Abstract: Let G be an abstract group acting on a space X. Let g be an element of G. It is a vague, but interesting problem what could be deduced about group properties of $g \in G$ from the dynamical properties of the action of g.

In the talk we would construct a natural cocycle on a class of groups of homeomorphisms. This class includes, among others, groups of Hamiltonian diffeomorphisms of symplectically aspherical manifolds and groups of homeomorphisms of a space preserving given first cohomology class. We will use this cocycle to prove that certain elements cannot be distorted (in other terms they have nonzero translation length). In particular, we reprove a theorem by Polterovich that groups of Hamiltonian diffeomorphisms of symplectically hyperbolic compact manifold do not contain such elements.

We will show that distorted elements must be "ergodic" (in a very vague sense), for example they cannot preserve different (in spirit of Nielsen) measures.

We will show how this idea was executed by Ch. Novak to show that the group of interval exchange transformations do not contain distorted elements.

As a side remark we will deduce that fundamental group of symplectically hyperbolic manifold cannot be amenable.