# Geometric and Asymptotic Group Theory II

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http://www.mat.univie.ac.at/∼dosaj/GGTWien/Course.html
Dienstag, 11:00–12:00, Raum D1.07 UZA 4

Blatt 2
**Free groups**

An *action* of a group $G$ on a set $X$ is a homomorphism $G \to \mathrm{Sym}(X)$.

(1) Prove the following Ping-pong Lemma.

Let $G$ be a group acting on a set $X$. Suppose there exist disjoint nonempty subsets $A^+, A^-, B^+, B^- \subset X$, and two elements $a, b$ of $G$ with the following properties:

a) $A^+ \cup A^- \cup B^+ \cup B^- \subsetneq X$;
b) $a(X - A^-) \subseteq A^+$, $a^{-1}(X - A^+) \subseteq A^-$;
b) $b(X - B^-) \subseteq B^+$, $b^{-1}(X - B^+) \subseteq B^-$.

Then $\langle a, b\rangle \leq G$ is a free subgroup generated by $a$ and $b$.

(2) Show that the fundamental group of a wedge (bouquet) of circles is free.

Hint: Consider the universal covering of the wedge of circles—a regular tree. Show, as in Exercise 6 from Blatt 1, that elements of the fundamental group correspond to "reduced" edge-paths in the tree.

(3) Show that every subgroup of a free group is free.

Hint: A subgroup of the fundamental group of $X$ is the fundamental group of some covering space of $X$.

(4) Show that the fundamental group of the torus with one hole is free. What about more holes or higer genus surface?