Geometric and Asymptotic Group Theory II

Damian Osajda
damian.osajda@univie.ac.at
http://www.mat.univie.ac.at/∼dosaj/GGTWien/Course.html
Dienstag, 11:00–12:00, Raum D1.07 UZA 4

Blatt 3

Examples of groups

(1) Prove that $S_3 = \langle a, b \mid a^2,\ b^2,\ (ab)^3 \rangle$.

Hint: Consider $a = (12), b = (13)$.

(2) Let $\varphi \colon F(s_1, s_2) \to \langle s_1 \rangle$, be given by $\varphi(s_1) = s_1$, $\varphi(s_2) = 1$. What is the basis and rank of $\mathrm{Ker}(\varphi)$?

(3) Show that $A \times B \ncong A * B$ for some groups A and B.

(4) Let $\varphi \colon A * B \to A \times B$ be such that $\varphi(a) = (a, 1)$, $\varphi(b) = (1, b)$, and $\varphi(aba^{-1}b^{-1}) = 1$, for $a \in A$, $b \in B$. Show that $\mathrm{Ker}(\varphi)$ is free and find its basis.

(5) Let a be the mirror symmetry of the Euclidean line $\mathbb{E}$ wrt the point 0. Let b be the symmetry of $\mathbb{E}$ wrt 1, Show that $\langle a, b \rangle \cong \langle a \rangle * \langle b \rangle$.

(6) Express the matrix $\begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix} \in SL(2, \mathbb{Z})$ as a product of the generators $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$.