Geometric and Asymptotic Group Theory I

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Dienstag, 11:00–12:00, Raum C2.07 UZA 4

Blatt 2
Cayley graphs

(1) Draw all the Cayley graphs of the cyclic group C_5 of order 5. Draw a few Cayley graphs of $\mathbb{Z}$ (integers with addition) and a few Cayley graphs of F_2 (the free group of rank 2).

(2) Show that $\mathcal{G}(G, S)$ is a tree iff $G = F(S)$.

(3) Draw a Cayley graph $\mathcal{G}$ of $\mathbb{Z}$ and a Cayley graph $\mathcal{G}'$ of $\mathbb{Z}^2$ having the following property. Combinatorial balls of radius 7 in $\mathcal{G}$ and $\mathcal{G}'$ are isomorphic.

(4) Does every Cayley graph have to be edge-transitive?

(5) Prove the Sabidussi Theorem: A graph Γ is a Cayley graph of a group G iff it admits a free transitive action of G by graph automorphisms.

(6) *Examples of groups.* Draw Cayley graphs of the following groups.
  (a) Baumslag-Solitar group $BS(2;1) = \langle a, b \mid ba^2b^{-1}a^{-1} \rangle$.
  (b) Heisenberg group $H_3(\mathbb{Z}) = \left\{ \begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \mid a, b, c \in \mathbb{Z} \right\}$.
  (c) The fundamental group of the surface of genus 2.
  (d) Right-angled Coxeter group $\langle a, b, c, d \mid a^2, b^2, c^2, d^2, [a, b], [c, d] \rangle$.
  (e) Right-angled Artin group $\langle a, b, c, d \mid [a, b], [c, d] \rangle$.

(7) Why is the Petersen graph not a Cayley graph?

Hint: Consider elements of order two in "the group".

(8) How to distinguish Cayley graphs of $\mathbb{Z}$ from the ones of $\mathbb{Z}^2$ and F_2?

Hint: Look at the graphs "from far away", i.e. consider asymptotic (or coarse, or large-scale geometry) properties of the graphs.