

Geometric and Asymptotic Group Theory I

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<http://www.mat.univie.ac.at/~dosaj/GGTWien/Course.html>

Dienstag, 11:00–12:00, Raum C2.07 UZA 4

Blatt 3

Group presentations

- (1) Show that if S is an infinite generating set for a finitely generated group G , then there exists a finite $S' \subset S$ generating G .
- (2) Let $\langle S \mid W \rangle$ and $\langle S \mid W' \rangle$ be two presentations of the same group G . Prove that if W is finite then there exists a finite $\overline{W} \subseteq W'$ such that $\langle S \mid \overline{W} \rangle$ is a presentation of G .
- (3) Show that the following groups have the corresponding presentations.
 - (a) Cyclic group of order 5; $\langle a \mid a^5 \rangle$.
 - (b) \mathbb{Z}^3 ; $\langle a, b, c \mid aba^{-1}b^{-1}, aca^{-1}c^{-1}, cbc^{-1}b^{-1} \rangle$.
 - (c) Dihedral group D_3 of order 6; $\langle a, b \mid a^3, b^2, abab \rangle$.
- (4) What is the order of the group $\langle a, b \mid a^2, b^2 \rangle$?
- (5) Show that the group $\langle a, b, c, d \mid abc^{-1}, bcd^{-1}, cda^{-1}, dab^{-1} \rangle$ is cyclic.
- (6) Show that if $G = \langle S \mid W \rangle$ and $G' = \langle S' \mid W' \rangle$ then $G \times G' = \langle S \cup S' \mid W \cup W' \cup \overline{W} \rangle$, where \overline{W} is the set of all words of the form $xyx^{-1}y^{-1}$, for $x \in S$ and $y \in S'$.