Geometric and Asymptotic Group Theory I

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http://www.mat.univie.ac.at/∼dosaj/GGTWien/Course.html

Dienstag, 11:00–12:00, Raum C2.07 UZA 4

Blatt 6
Random groups

(1) Under the assumptions of the Big Face Theorem show that:
  (a) $\exists \beta \ |\partial D| \geq \beta \sum_i |\partial D_i|$,
      where D_i are faces in van Kampen diagram D.
  (b) In fact, we can take $\beta = (1 - 6\lambda)$, for $\lambda \leq \frac{1}{6}$ from $C'(\lambda)$.

(2) Prove that in a set of 2^{dl} randomly chosen words of length l in letters $a^{\pm 1}, b^{\pm 1}$, there will be probably some word beginning with dl letters a (as $l \to \infty$ and d is a fixed density parameter $0 < d < 1$).

(3) In a set of $(2m - 1)^{dl}$ of randomly chosen words of length l in $2m$ letters $s_1^{\pm 1}, ..., s_m^{\pm 1}$ there will be probably words having the same $(2dl)$ letters, but no more.

(4) Prove the Probabilistic Pigeon-hole Principle: Let $\epsilon > 0$. Put $N^{\frac{1}{2} + \epsilon}$ pigeons uniformly at random among N pigeon-holes. Then there are two pigeons in the same hole with probability $\to 1$ as $N \to \infty$, and this happens arbitrarily many times with growing N.