Geometric and Asymptotic Group Theory II

Damian Osajda
damian.osajda@univie.ac.at
http://www.mat.univie.ac.at/∼dosaj/GGTWien/Course.html
Dienstag, 11:00–12:00, Raum C2.07 UZA 4

Blatt 3
Group actions

(1) Show that every group acts freely and transitively on itself by left multiplications.

(2) For a prime number p, let $\mathbb{Z}_p$ act on a set X with $p \nmid |X|$. Show that there is a fixed point for this action, i.e. a point $x \in X$ such that $gx = x$, for every $g \in \mathbb{Z}_p$.

(3) Prove the following *Burnside's Lemma*.

Let G act on a set X. Then $|G||X/G| = \sum_{g \in G} |X^g|$.

(4) Prove the following *Ping-pong Lemma*.

Let G be a group acting on a set X. Suppose there exist disjoint nonempty subsets $A^+, A^-, B^+, B^- \subset X$, and two elements a, b of G with the following properties:

a) $A^+ \cup A^- \cup B^+ \cup B^- \subsetneq X$;
b) $a(X - A^-) \subseteq A^+$, $a^{-1}(X - A^+) \subseteq A^-$;
b) $b(X - B^-) \subseteq B^+$, $b^{-1}(X - B^+) \subseteq B^-$.

Then $\langle a, b\rangle \leq G$ is a free subgroup generated by a and b.

(5) Let G be a finite group. Prove that every G-action by automorphisms on a tree has a fixed point. Show that the set of fixed points is contractible.