Geometric and Asymptotic Group Theory II (Prof. G. Arzhantseva)
Summer 2012

Exam questions

(1) Group presentations. Free groups, Baumslag-Solitar groups, HNN-extensions: definition, basic properties, the Cayley graph, examples, Ping-Pong lemma.

(2) Hopfian groups: definition, (non)-examples, Malcev's theorem.

(3) Residually finite groups: equivalent definitions, (non)-examples, free group of finite rank is residually finite (2 proofs).

(4) Britton's lemma for Baumslag-Solitar group BS(1,2).

(5) Residually finite and Hopfian Baumslag-Solitar groups: proof of Case 1°. Examples of (non)-metabelian groups.

(6) Residually finite and Hopfian Baumslag-Solitar groups: proof of Case 2°. Free product of residually finite groups is residually finite (proof).

(7) Residually finite and Hopfian Baumslag-Solitar groups: proof of Case 3°. Every subgroup of a residually finite group is residually finite (proof).

(8) Residually finite and Hopfian Baumslag-Solitar groups: proof of Case 4°.

(9) Amenability: definition via Folner constants, (non)-examples. Free group is not amenable (proof).

(10) Amenability: definition via Folner constants, independence of the choice of generating sets and of the boundary.

(11) Equivalent definitions of amenability: proofs of (1)$\rightarrow$(2), (1)$\rightarrow$(3).

(12) Equivalent definitions of amenability: proofs of (2)$\rightarrow$(3).

(13) Equivalent definitions of amenability: proofs of (3)$\rightarrow$(1).

(14) (Non)-amenable Baumslag-Solitar groups. Metabelian groups are amenable (proof).

(15) The space of marked groups: definition, basic properties, and examples of converging sequences.

(16) Weak amenability of Baumslag-Solitar groups.

(17) Bass-Serre theory: Baumslag-Solitar groups BS(2,3).

(18) Bass-Serre theory: HNN-extensions.