Geometric and Asymptotic Group Theory II (Prof. G. Arzhantseva)
Summer semester -- 2013

Exam questions

(1) Free groups: definitions (universal and construction), basic properties, the Cayley graph, rank, examples, subgroups of free groups.

(2) Graphs representing subgroups of free groups: definition, basic properties, examples, Stallings foldings, free and Nielsen-reduced free bases.

(3) Fully residually free groups: definition, basic properties, and examples.

(4) Random subgroups of free groups: few-relator model and Stallings graph model. Generic finitely presented groups, examples of generic properties.

(5) Classical small cancellation condition C'(1/6): definition, examples, genericity. The Greendlinger lemma (formulation).

(6) Generic Hanna Neumann conjecture (strong form, with proof).

(7) Mineyev's proof of Hanna Neumann Conjecture: basic steps.

(8) Generic finitely presented group in the Stallings model is trivial (sketch of the proof).

(9) C*-algebras: definition and (counter)-examples; Gelfand's theorem (formulation).

(10) Gelfand-Neumark-Segal's theorem (formulation); The reduced C*-algebra of a group: definition and examples.

(11) The full C*-algebra of a group: definition, examples, the universal property, non-simplicity, the existence of a character.

(12) Theorem on the equivalent characterizations of amenability: proof of (1) -> (2); The Folner condition: examples, the subgroup theorem.

(13) Theorem on the equivalent characterizations of amenability: proof of (2) -> (3); Fell's absorption principle.

(14) Theorem on the equivalent characterizations of amenability: proof of (3) -> (4); State (definition and examples; tracial and faithful states), proof of Lemma 1 and of Lemma 2.

(15) Theorem on the equivalent characterizations of amenability: proof of (3) -> (4); Character (definition and examples), proof of Lemma 3 and of Lemma 4.

(16) Theorem on the equivalent characterizations of amenability: proof of (3) -> (4);

(17) Banach-Tarski Paradox: proof and relations with amenability, Tarski's theorem (without proof).

(18) Simplicity of C*-algebras: the Dixmier property implies simplicity; A faithful tracial state implies its uniqueness.

(19) Simplicity of the reduced C*-algebra of the free group: Lemma and Corollary.

(20) Simplicity of the reduced C*-algebra of the free group: Theorem.

(21) Weak amenability properties: definitions and examples. Amenability implies coarse amenability; a-T-menability implies coarse embeddings.

(22) Weak amenability properties: amenable implies a-T-menable.

(23) Weak amenability properties: coarse amenable implies coarse embeddable.