Geometric and Asymptotic Group Theory

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Dienstag, 11:00–12:00, Seminarraum 8 Oskar-Morgenstern-Platz 1 2.Stock

Blatt 3
Residual finiteness

(1) Show that for a group G the following conditions are equivalent — a group satisfying them is *residually finite*.

  (a) For every element $g \neq 1_G$ in G there exists a homomorphism $\varphi\colon G \to F$ into some finite group F, such that $\varphi(g) \neq 1_F$.
  (b) For every element $g \neq 1_G$ in G, there exists a finite index subgroup $K \leq G$ with $g \notin K$.
  (c) For every finite set A of nontrivial elements in G, there exists a homomorphism $\varphi\colon G \to F$ into some finite group F, such that $\varphi(g) \neq 1_F$, for every $g \in A$.
  (d) The intersection of all (normal) subgroups of G of finite index is trivial.
  (e) Let $G = \pi_1(X, x_0)$. For every homotopically non-trivial loop γ in (X, x_0) there is a finite covering $p\colon \widetilde{X} \to X$ such that γ does not lift up to a loop in $\widetilde{X}$.

(2) Show that $\mathbb{Z}$ and $\mathbb{Z}^2$ are residually finite.

(3) Show that every finitely generated residually finite group is Hopfian.

(4) Show that the group $(\mathbb{Q}, +)$ is Hopfian, and $(\mathbb{R}, +)$ is not.

(5) Show that $\mathrm{Aut}(\mathbb{F}_{\mathbb{N}})$ contains a simple subgroup.