Geometric and Asymptotic Group Theory

Damian Osajda
damian.osajda@univie.ac.at
http://www.mat.univie.ac.at/∼dosaj/GGTWien/Course.html
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Blatt 5

Ends of groups

A map $f\colon (X, d_X) \to (Y, d_Y)$ between metric spaces is a (K, L)–*quasi-isometric embedding* if, for every $x, y \in X$ the following inequalities are satisfied

$$K^{-1} d_Y(f(x), f(y)) - L \leqslant d_X(x, y) \leqslant K d_Y(f(x), f(y)) + L.$$

A (K, L)–quasi-isometric embedding f is (K, L)–*quasi-isometry* if every point in Y is at distance at most L from some point in the image $f(X)$.

(1) Show that for any two finite generating sets S, S' of a group G, the identity map on G is a quasi-isometry between (G, d_S) and $(G, d_{S'})$ (word metrics wrt S and S').

(2) A geodesic metric space *has one end* if the complement of any bounded subset has one unbounded component. Show that if geodesic metric spaces X, Y are quasi-isometric then X has one end iff Y has one end. Define one-endedness for a finitely generated group.

(3) Which of the following groups have one end: $\mathbb{Z}/2\mathbb{Z}$, $\mathbb{Z}$, $\mathbb{Z}^2$, $\mathbb{F}_2$?