Representation Theory of Groups - Blatt 4

11:30-12:15, Seminarraum 9, Oskar-Morgenstern-Platz 1, 2.Stock

http://www.mat.univie.ac.at/~gagt/rep_theory2016

Goulnara Arzhantseva
goulnara.arzhantseva@univie.ac.at

Martin Finn-Sell
martin.finn-sell@univie.ac.at

Question 1. Prove that every finite simple group G has a faithful simple $\mathbb{C}G$-module.

Question 2. Let k be a field, $V \subset k^n$ be a vector subspace and let $M_n(k)$ denote the set of $n \times n$ matrices over k. Define M_V to be the set of all matrices in $M_n(k)$ whose rows, when thought of as vectors in k^n, are contained in V. Show:

a) M_V is an invariant subspace of the left regular $M_n(k)$-module of dimension $n \dim_k(V)$;

b) Every invariant subspace of the left regular $M_n(k)$-module is of the form M_V for some subspace V of k^n;

c) M_V is simple if and only if V is one dimensional;

d) M_V is isomorphic to M_W as a $M_n(k)$-module if and only if the two subspaces V and W have the same dimension.

Question 3. Let $A \in M_n(k)$, where k is an algebraically closed field. Suppose that $A^m = 1$ for m not divisible by the characteristic of k. Show that A is diagonalisable.

Question 4. (Needs a little more commutative algebra) Let G be a finite group and let k be a field. Show that every module over kG is projective if and only if $\mathrm{char}(k)$ does not divide the order of the group G.