Representation Theory of Groups - Blatt 2

11:30-12:15, Seminarraum 9, Oskar-Morgenstern-Platz 1, 2.Stock

http://www.mat.univie.ac.at/~gagt/rep_theory2017

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Question 1. Let $G = C_2 \times C_4 \times C_5$, where C_n denotes the cyclic group of order n. Calculate all the irreducible representations of G.

Question 2. Construct two inequivalent irreducible representations of the restricted wreath product $S_3 \wr C_2$.

Question 3. Let G be a group and H a subgroup of G. Show that any irreducible representation of G is contained in some induced irreducible representation of H.

Question 4. (Maschke's theorem and characteristic p) Let $G = C_p = \langle g | g^n = 1 \rangle$ and consider the map $\rho : G \to GL_2(\mathbb{F}_p)$ given by:

$$g \mapsto \begin{pmatrix} 1 & g \\ 0 & 1 \end{pmatrix}.$$

Check that this defines a representation of G, and show that Maschke's Theorem fails for the 1-dimensional subspace of $\mathbb{F}_p$ fixed by ρ.

Question 5. Show that if G has a unique, minimal normal subgroup N, then G has a faithful irreducible representation.